Algebra Checkpoint ✓

Check your knowledge of factoring polynomial expressions by doing the following problems. You can review these topics in **Algebra Toolkit B.2** on page T33.

1. Factor out the common factor.
(a) $6x^2 + 3$ **(b)** $r^2 - 3r$ **(c)** $t^4 + 4t^2$ **(d)** $2y^3 - 6y^2 + 4y$

2. Factor the quadratic.
(a) $x^2 - 81$ **(b)** $25m^2 + 10m + 1$ **(c)** $t^2 + 3t - 28$ **(d)** $3n^2 - n - 2$

3. Factor the expression completely.
(a) $3t^5 - 12t^3$ **(b)** $w^4 - 16$ **(c)** $x^5 + 3x^4 - 28x^3$ **(d)** $r^6 - 12r^3 + 36$

4. Factor the expression by grouping terms.
(a) $x^3 + 3x^2 + x + 3$ **(b)** $7z^5 - 14z^3 + 5z^2 - 10$

6.3 Exercises

1. (a) A polynomial function is a sum of ________ functions.
(b) The polynomial $P(x) = 3x^4 - 2x^3 + 4$ is the sum of the power functions $y =$ ________, $y =$ ________, and $y =$ ________. The leading term of P is ________, and the degree of P is ________.

2. (a) To find the zeros of a polynomial function, we first ________ the polynomial and then apply the Zero-Product Property. The zeros of a polynomial are the ____-intercepts of the graph of the polynomial.
(b) The polynomial function $f(x) = (x - 2)(x + 1)(x - 3)$ is in factored form. The zeros of f are ____, ____, ____. The x-intercepts of the graph of f are ____, ____, ____.

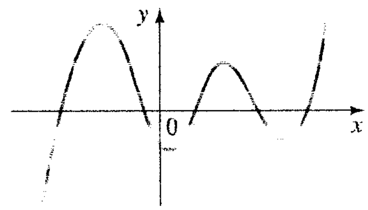
3. Between consecutive x-intercepts the values of a polynomial must always be ______________ or always be ______________. To decide which sign is the appropriate one, we use ________ points.

4. Every nonconstant polynomial has one of the following end behaviors:
(i) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(ii) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(iii) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(iv) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

For each polynomial, choose the appropriate description of its end behavior from the list above.
(a) $y = x^3 - 8x^2 + 2x - 15$; end behavior: ______________________.
(b) $y = -x^3 + 8x^2 - 2x + 15$; end behavior: ______________________.
(c) $y = 5x^6 + 3x^3 - 9x - 12$; end behavior: ______________________.
(d) $y = -5x^6 - 3x^3 + 9x + 12$; end behavior: ______________________.

Think About It

5. Can the polynomial whose graph is shown have a degree that is even?



6. A function f is *odd* if $f(-x) = -f(x)$ or *even* if $f(-x) = f(x)$.
- **(a)** Explain why a power function that contains only odd powers of x is an odd function.
- **(b)** Explain why a power function that contains only even powers of x is an even function.
- **(c)** Explain why a power function that contains both odd and even powers of x is neither an odd nor an even function

7–8 ■ Graph the given polynomial function by expressing it as a sum of power functions, and then using graphical addition.

7. $f(x) = -x^3 + 2x^2 + 5$ **8.** $f(x) = x^4 + 2x - 1$

9. Let $f(x) = x^3 + x^2 - x - 1$.
- **(a)** Show that $f(x) = (x + 1)^2(x - 1)$.
- **(b)** Find the x-intercepts of the graph of f.
- **(c)** Find the sign of f on each of the intervals determined by the x-intercepts.
- **(d)** Sketch a graph of f.

10. Let $f(x) = x^3 + 3x^2 - 4x - 12$.
- **(a)** Show that $f(x) = (x + 3)(x - 2)(x + 2)$.
- **(b)** Find the x-intercepts of the graph of f.
- **(c)** Find the sign of f on each of the intervals determined by the x-intercepts.
- **(d)** Sketch a graph of f.

11–18 ■ Sketch a graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

11. $P(x) = (x - 1)(x + 2)$ **12.** $P(x) = (x - 1)(x + 1)(x - 2)$

13. $P(x) = x(x - 3)(x + 2)$ **14.** $P(x) = (2x - 1)(x + 1)(x + 3)$

15. $P(x) = (x - 3)(x + 2)(3x - 2)$ **16.** $P(x) = \frac{1}{5}x(x - 5)^2$

17. $P(x) = (x - 1)^2(x - 3)$ **18.** $P(x) = (x - 3)^2(x + 1)^2$

19–26 ■ A polynomial function P is given.
- **(a)** Express the function P in factored form.
- **(b)** Sketch a graph of P.

19. $P(x) = x^3 - x^2 - 6x$ **20.** $P(x) = x^3 + 2x^2 - 8x$

21. $P(x) = -x^3 + x^2 + 12x$ **22.** $P(x) = -2x^3 - x^2 + x$

23. $P(x) = x^4 - 3x^3 + 2x^2$ **24.** $P(x) = x^5 - 9x^3$

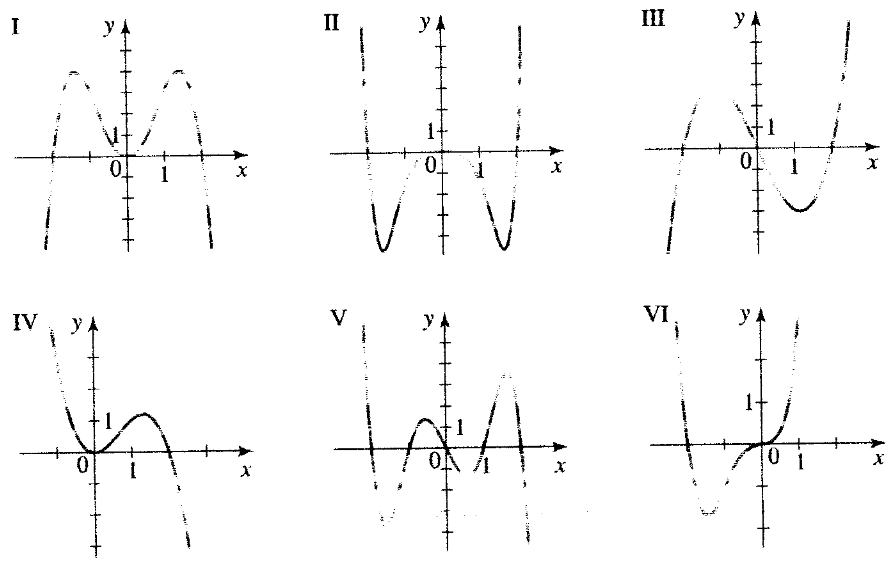
25. $P(x) = x^4 - 3x^2 - 4$ **26.** $P(x) = x^6 - 2x^3 + 1$

27–32 ■ Determine the end behavior of P. Compare the graphs of P and Q on large and small viewing rectangles as in Example 5.

27. $P(x) = 3x^3 - x^2 + 5x + 1, \quad Q(x) = 3x^3$
28. $P(x) = -\frac{1}{8}x^3 + \frac{1}{4}x^2 + 12x, \quad Q(x) = -\frac{1}{8}x^3$
29. $P(x) = x^4 - 7x^2 + 5x + 5, \quad Q(x) = x^4$
30. $P(x) = -x^5 + 2x^2 + x, \quad Q(x) = -x^5$
31. $P(x) = x^{11} - 9x^9, \quad Q(x) = x^{11}$
32. $P(x) = 2x^2 - x^{12}, \quad Q(x) = -x^{12}$

33–38 ■ Match the polynomial function with one of the graphs I–VI. Give reasons for your choice.

33. $P(x) = x(x^2 - 4)$
34. $Q(x) = -x^2(x^2 - 4)$
35. $R(x) = -x^5 + 5x^3 - 4x$
36. $S(x) = \frac{1}{2}x^6 - 2x^4$
37. $T(x) = x^4 + 2x^3$
38. $U(x) = -x^3 + 2x^2$



39–42 ■ A polynomial function P is given.

(a) Graph the polynomial P in the given viewing rectangle.
(b) Find all the local maxima and minima of P.
(c) Find all solutions of the equation $P(x) = 0$.

39. $P(x) = x^3 - 3x^2 - 4x + 12$; $[-4, 4]$ by $[-15, 15]$
40. $P(x) = x^4 - 5x^2 + 4$; $[-4, 4]$ by $[-30, 30]$
41. $P(x) = x^5 - 5x^2 + 6$; $[-3, 3]$ by $[-5, 10]$
42. $P(x) = 2x^3 - 8x^2 + 9x - 9$; $[-4, 6]$ by $[-40, 40]$

CONTEXTS

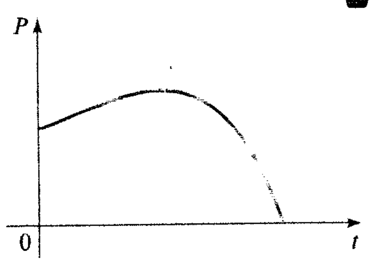
43. **Market Research** A market analyst working for a small appliance manufacturer finds that if the firm produces and sells x blenders annually, a model for the total profit (in dollars) is

$$P(x) = 8x + 0.3x^2 - 0.001x^3 - 372$$

Graph the function P in an appropriate viewing rectangle, and use the graph to answer the following questions.

(a) When just a few blenders are manufactured, the firm loses money (profit is negative). (For example, $P(10) = -263.3$, so the firm loses \$263.30 if it produces and sells only 10 blenders.) How many blenders must the firm produce to break even?

(b) Does profit increase indefinitely as more blenders are produced and sold? If not, what is the largest possible profit the firm could have?



44. Population Change The rabbit population on a small island is observed to be modeled by the function.

$$P(x) = 120t - 0.4t^4 + 1000$$

where t is the time (in months) since observations of the island began. Graph the function P in an appropriate viewing rectangle, and use the graph to answer the following questions.

(a) When is the maximum population attained, and what is that maximum population?

(b) When does the rabbit population disappear from the island?

45. Depth of Snowfall Snow began falling at noon on Sunday. The amount of snow on the ground at a certain location at time t was modeled by the function.

$$h(t) = 11.60t - 12.41t^2 + 6.20t^3 - 1.58t^4 + 0.20t^5 - 0.01t^6$$

where t is measured in days from the start of the snowfall and $h(t)$ is the depth of snow in inches. Graph this function in an appropriate viewing rectangle, and use your graph to answer the following questions.

(a) What happened shortly after noon on Tuesday?

(b) Were there ever more than 5 inches of snow on the ground? If so, on what day(s)?

(c) On what day and at what time (to the nearest hour) did the snow disappear completely?

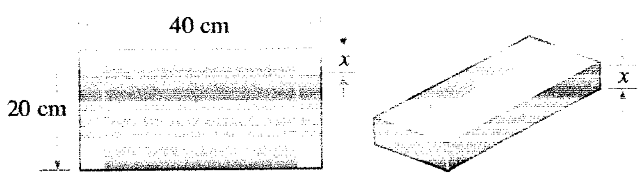
46. Volume of a Box An open box is to be constructed from a piece of cardboard 20 cm by 40 cm by cutting squares of side length x from each corner and folding up the sides, as shown in the figure.

(a) Show that the volume of the open box is given by the function

$$V(x) = 4x^3 - 120x^2 + 800x$$

(b) What is the domain of V? (Use the fact that length and volume must be positive.)

(c) Draw a graph of the function V, and use it to estimate the maximum volume for such a box.



47. Volume of a Box A cardboard box has a square base, with each edge of the base having length x inches, as shown in the figure on the following page. The total length of all 12 edges of the box is 144 inches.

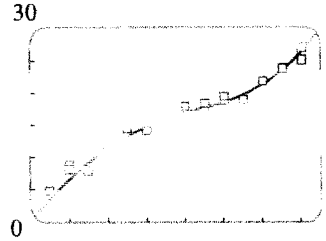
(a) Show that the volume of the open box is given by the function

$$V(x) = 2x^2(18 - x)$$

Here, x is age in years and y is length in inches. A scatter plot of the data together with a graph of the cubic polynomial model is shown in Figure 6(b).

```
CubicReg
 y=ax³+bx²+cx+d
 a=.0154911306
 b=-.372473323
 c=3.94608636
 d=1.21080418
```

(a) CubicReg output



(b) Scatter plot and model

figure **6** Cubic polynomial model for length-at-age data

(b) We replace x by 5 in the model.

$$y = 0.0155x^3 - 0.372x^2 + 3.95x + 1.21 \qquad \text{Model}$$
$$y = 0.0155(5)^3 - 0.372(5)^2 + 3.95(5) + 1.21$$
$$y \approx 13.6$$

So a 5-year-old rock bass would be approximately 13.6 inches long.

(c) Moving the cursor along the graph of the polynomial in Figure 6(b), we find that y is 20 when x is approximately 10.8. So the fish is about 10.8 years old.

NOW TRY EXERCISE 21

6.4 Exercises

CONCEPTS

Fundamentals

1. **(a)** When modeling data, we make a ________________ plot to help us visually determine whether a line or some other curve is appropriate for modeling the data.
   (b) If the y-values of a set of data increase rapidly, then an exponential function or a ________________ function may be appropriate for modeling the data.

2. To determine whether an exponential function or a power function is appropriate for modeling a set of data, we make a semi-log plot and a log-log plot of the data.
   (a) If the semi-log plot lies approximately along a line, then a ________________ model is appropriate.
   (b) If the log-log plot lies approximately along a line, then a ________________ model is appropriate.

Think About It

3–4 The following data are obtained from the function $f(x) = 5x^3$.

x	1	2	3	4	5
$f(x)$	5	40	135	320	625

3. If you use your calculator to find the power function that best fits the data, what function would you expect to get? Try it.

4. If you use your calculator to find the cubic polynomial that best fits the data, what function would you expect to get? Try it.

5–8 ■ A set of data is given.

(a) Use your calculator to find the indicated model for the data.

(b) Graph a scatter plot of the data along with your model. Does the model fit the data?

5. Power:

x	$f(x)$
0.1	10
1.0	254
1.4	298
1.9	480
2.0	584
2.6	698
3.1	822
3.7	1141

6. Linear:

x	$f(x)$
6	314
9	401
10	869
22	1126
29	1352
37	2251
62	3112
114	3551

7. Cubic:

x	$f(x)$
0.6	24.2
1.5	32.8
2.0	34.0
2.4	33.2
3.9	24.9
4.8	20.6
5.2	22.7
5.6	25.8

8. Exponential:

x	$f(x)$
0.5	30
1.1	51
2.3	72
3.0	89
3.2	115
4.1	168
4.9	206
5.5	342

Data for Exercise 9

x	$f(x)$
1.0	4
1.5	16.5
2.0	45.3
2.3	73.8
3.0	187.1
3.5	320.9
3.7	389.7
4.1	558.2
4.4	714.7
5.0	1118.0

Data for Exercise 10

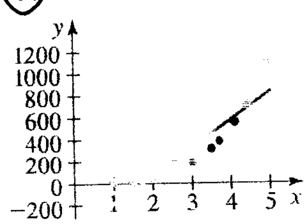
x	$f(x)$
0.3	1.50
1.0	1.72
1.2	1.79
1.7	1.95
2.1	2.08
2.4	2.17
2.8	2.30
3.0	2.37
3.4	2.50
4.0	2.69

9–10 ■ A scatter plot, a semi-log plot, and a log-log plot for the set of data given in the margin are shown below. Each graph also shows the regression line.

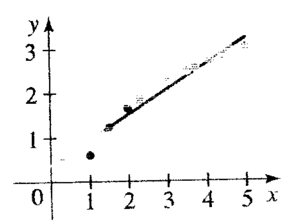
(a) Is a linear, exponential, or power function more appropriate for modeling the data?

(b) Find the model that is most appropriate, and graph the scatter plot along with your model. Does the model fit the data?

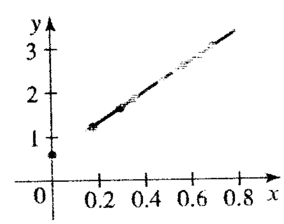
9.



Scatter plot

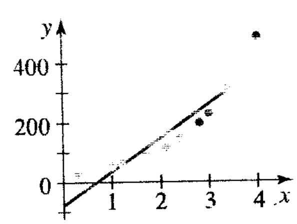


Semi-log plot

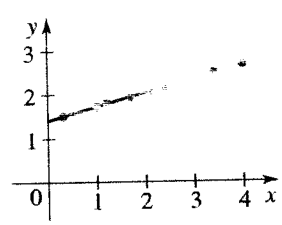


Log-log plot

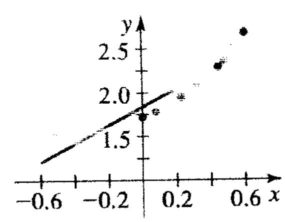
10.



Scatter plot



Semi-log plot



Log-log plot

11–14 ■ Data points (x, y) are given (see the next page).

(a) Draw a scatter plot of the data.

(b) Make semi-log and log-log plots of the data.

(c) Is a linear, power, or exponential function appropriate for modeling these data?

(d) Find an appropriate model for the data, and then graph the model together with a scatter plot of the data.

11.

x	y
2	0.08
4	0.12
6	0.18
8	0.25
10	0.36
12	0.52
14	0.73
16	1.06

12.

x	y
10	29
20	82
30	151
40	235
50	330
60	430
70	546
80	669
90	797
100	935

13.

x	y
1	1.5
2	5.1
3	11.3
4	20.1
5	32.4
6	45.5
7	62.1
8	81.1
9	102.8
10	127.0

14.

x	y
3	0.003
6	0.012
9	0.042
12	0.153
15	0.551
18	1.992
21	7.239
24	26.279

15. A Falling Ball In a physics experiment a lead ball is dropped from a height of 5 meters. The students record the distance the ball has fallen every one-tenth of a second. (This can be done by using a camera and a strobe light.)

(a) Make a scatter plot of the data.

(b) Use a graphing calculator to find a power function of the form $d = at^b$ that models the distance d that the ball has fallen after t seconds.

(c) Draw a graph of the function you found and the scatter plot on the same graph. How well does the model fit the data?

(d) Use your model to predict how far a dropped ball would fall in 3 seconds.

Time (s)	Distance (m)
0.1	0.048
0.2	0.197
0.3	0.441
0.4	0.882
0.5	1.227
0.6	1.765
0.7	2.401
0.8	3.136
0.9	3.969
1.0	4.902

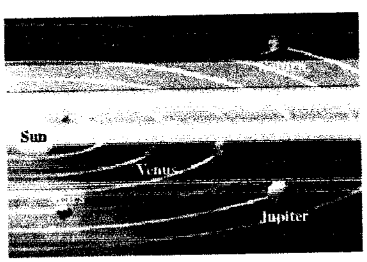
16. Kepler's Law for Periods of the Planets For each planet, its mean distance d from the sun [in astronomical units (AU)] and its period T (in years) are given in the table. (Although Pluto is no longer considered a planet, we include it in the table because the distance-period relationship we study here applies to any body orbiting the sun.)

(a) Make a scatter plot of the data. Is a linear model appropriate?

(b) Find a power function that models the data.

(c) Draw a graph of the function you found and the scatter plot on the same graph. How well does the model fit the data?

(d) Use the model that you found to calculate the period of an asteroid whose mean distance from the sun is 5 AU.



Distance and period

Planet	d (AU)	T (yr)
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.000	1.000
Mars	1.523	1.881
Jupiter	5.203	11.861
Saturn	9.541	29.457
Uranus	19.190	84.008
Neptune	30.086	164.784
Pluto	39.507	248.350

Year	Lead emissions
1970	199.1
1975	143.8
1980	68.0
1985	18.3
1988	5.9
1989	5.5
1990	5.1
1991	4.5
1992	4.7

17. Lead Emissions The table at the left gives U.S. lead emissions into the environment in millions of metric tons for 1970–1992.

(a) Make a scatter plot of the data.

(b) Find an exponential model for these data. (Use $t = 0$ for the year 1970.)

(c) Find a fourth-degree polynomial that models the data.

(d) Which of these curves gives a better model for the data? Use graphs of the two models to decide.

Tree species of the Pasoh Forest of Malaysia

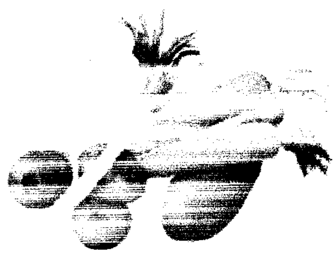
Area (m^2)	Observed number of species
3.81	3
7.63	3
15.26	12
30.52	13
61.04	31
122.07	70
244.14	112
488.28	134
976.56	236

20. Biodiversity To test for the biodiversity of trees in a tropical rain forest, biologists collected data in the Pasoh Forest Reserve of Malaysia. The table in the margin shows the number of tree species S found for a given area A in the rain forest.*

(a) Use a graphing calculator to find a power function of the form $S = aA^b$ that models the number of tree species S that are in an area of size A. Then find an exponential function of the form $S = ab^A$ to model the data.

(b) Make a scatter plot of the data, and graph both functions that you found in part (a) on your scatter plot.

(c) Which of these curves gives a better model for the data? Use graphs of the two models to decide.



21. How Fast Can You Name Your Favorite Things? If you are asked to make a list of objects in a certain category, the speed with which you can list them follows a predictable pattern. For example, if you try to name as many vegetables as you can, you'll probably think of several right away—for example, carrots, peas, beans, corn, and so on. Then after a pause you might think of ones you eat less frequently—perhaps zucchini, eggplant, and asparagus. Finally, more exotic vegetables might come to mind—artichokes, jicama, bok choy, and the like. A psychologist performs this experiment on a number of subjects. The table below gives the average number of vegetables that the subjects named within a given number of seconds.

(a) Find the cubic polynomial that best fits the data.

(b) Draw a graph of the polynomial from part (a) together with a scatter plot of the data.

(c) Use your result from part (b) to estimate the number of vegetables that subjects would be able to name in 40 seconds.

(d) According to the model, how long (to the nearest 0.1 second) would it take a person to name five vegetables?

Seconds	Number of vegetables
1	2
2	6
5	10
10	12
15	14
20	15
25	18
30	21

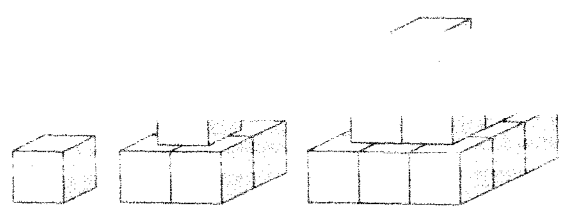
22. Polynomial Pattern The figure shows a sequence of pyramids made of cubic blocks.

(a) Complete the table for the number of blocks in a pyramid with n layers.

(b) Find the cubic polynomial P that best fits the data you obtained.

(c) Compare the values $P(1), P(2), P(3), \ldots$ with the data in the table. Does the polynomial you found model the data exactly?

Layers	1	2	3	4	5	6
Blocks	1	5	14			



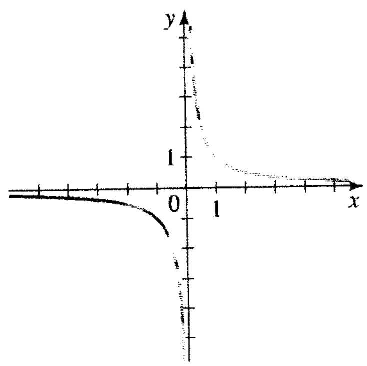
*K. M. Kochummen, J.V. LaFrankie, and N. Manokaran, "Floristic Composition of Pasoh Forest Reserve, a Lowland Rain Forest in Peninsular Malaysia," *Journal of Tropical Forest Science*, 3:1–13, 1991.

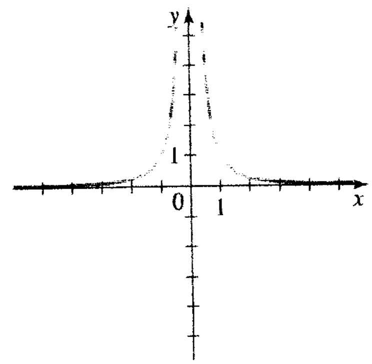
6.5 Exercises

CONCEPTS

Fundamentals

1. Graphs of $y = x^{-1}$ and $y = x^{-2}$ are shown. Identify each graph.





$y =$ ________ $y =$ ________

2. (a) If the quantities x and y are related by the equation $y = 7/x^5$, then we say that y is ________ proportional to the ________ power of x and the constant of proportionality is ________.

   (b) If y is inversely proportional to the third power of x and if the constant of proportionality is 4, then x and y are related by the equation $y = \square \cdot \dfrac{1}{x^{\square}}$.

Think About It

3. Graph the function $y = x^{-1}$ in the viewing rectangle $[-1, 1]$ by $[-1, 1]$. Explain why you get a blank screen.

4. Suppose that a skunk is perched at the top of a tall flag pole. Do you think that the intensity of its smell is inversely proportional to the square of the distance from the skunk? (Read the explanation of the geometry of inverse square laws on page 531.)

SKILLS

5–8 ■ A power function f is given.

(a) Complete each table for the values of the function.

(b) Describe the behavior of the function near its vertical asymptote, based on Tables 1 and 2.

(c) Determine the behavior of the function near its horizontal asymptote, based on Tables 3 and 4.

table **1**

x	$f(x)$
−0.1	
−0.01	
−0.001	
−0.00001	

table **2**

x	$f(x)$
0.1	
0.01	
0.001	
0.00001	

table **3**

x	$f(x)$
10	
50	
100	
100,000	

table **4**

x	$f(x)$
−10	
−50	
−100	
−100,000	

5. $f(x) = x^{-3}$ 6. $f(x) = x^{-5}$ 7. $f(x) = x^{-4}$ 8. $f(x) = x^{-6}$

9–12 ▪ Graph the family of functions in the same viewing rectangle, using the given values of c. Explain how changing the value of c affects the graph.

9. $f(x) = x^{-c}$; $c = 1, 3, 5, 7$

10. $g(x) = x^{-c}$; $c = 2, 4, 6, 8$

11. $F(x) = cx^{-1}$; $c = 1, 2, 5, \frac{1}{2}$

12. $G(x) = x^{-2} + c$; $c = -1, 0, 1, 2$

13–20 ▪ Sketch a graph of the given function.

13. $f(x) = 3x^{-1}$

14. $f(x) = 10x^{-2}$

15. $f(x) = 5x^{-4}$

16. $f(x) = 2x^{-7}$

17. $f(x) = \dfrac{4}{x^2}$

18. $f(x) = \dfrac{6}{x}$

19. $f(x) = \dfrac{1}{x} + 1$

20. $f(x) = \dfrac{1}{x^2} - 1$

21–24 ▪ Write an equation that expresses the statement.

21. P is inversely proportional to T.

22. R is inversely proportional to the third power of e.

23. y is inversely proportional to the square root of t.

24. E is inversely proportional to the fourth root of t.

25–28 ▪ Express the statement as an equation. Use the given information to find the constant of proportionality.

25. z is inversely proportional to t. If t is 3, then z is 2.

26. R is inversely proportional to s. If s is 4, then R is 3.

27. s is inversely proportional to the square root of t. If t is 9, then s is 15.

28. W is inversely proportional to the square of r. If r is 6, then W is 10.

29–30 ▪ Use the given information to solve the problem.

29. V is inversely proportional to the cube of x. If the constant of proportionality k is 1.5, then find V when x is 2.

30. C is inversely proportional to the square of t. If the constant of proportionality k is 5, then find C when t is 7.

31. Boyle's Law The pressure P of a sample of oxygen gas that is compressed at a constant temperature is inversely proportional to the volume V of gas.

(a) Find the constant of proportionality if a sample of oxygen gas that occupies $0.671\ \text{m}^3$ exerts a pressure of 39 kPa at a temperature of 293 K (absolute temperature measured on the Kelvin scale). Write the equation that expresses the inverse proportionality.

(b) If the sample expands to a volume of $0.916\ \text{m}^3$, find the new pressure.

32. Skidding in a Curve A car weighing 1000 kilograms is traveling at a speed of 15 meters per second on a curve that forms a circular arc. The centripetal force F that pushes the car outward is inversely proportional to the radius r (measured in meters) of the curve.

(a) If the curve has a radius of 90 meters, then the force required to keep the car from skidding is 2500 newtons. Write an equation that expresses the inverse proportionality.

(b) The car will skid if the centripetal force is greater than the frictional force holding the tires to the road. For this road the maximum frictional force is 5880 newtons. What is the radius of the tightest curve the car can maneuver before it starts slipping?

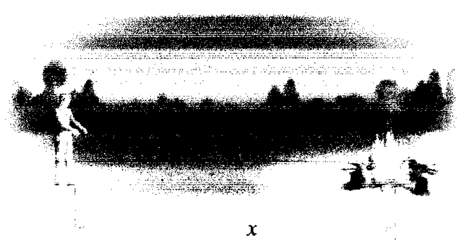
33. **Loudness of Sound** The loudness L of a sound measured in decibels (dB) is inversely proportional to the square of the distance d from the source of the sound.
   (a) A person who is 10 ft from a lawn mower experiences a sound level of 70 dB. Find the constant of proportionality, and write the equation that expresses the inverse proportionality.
   (b) How loud is the lawn mower when the person is 100 ft away?

34. **Electrical Resistance** The resistance R of a wire of a fixed length is inversely proportional to the square of its diameter.
   (a) A wire that is 1.2 meters long and 0.005 meter in diameter has a resistance of 140 ohms. Find the constant of proportionality, and write the equation that expresses the inverse proportionality.
   (b) Find the resistance of a wire made of the same material and of the same length as the wire in part (a) but with a diameter of 0.008 meter.

35. **Growing Cabbages** In the short growing season of the Canadian arctic territory of Nunavut, some gardeners find it possible to grow gigantic cabbages in the midnight sun. Assume that the cabbages receive a constant amount of nutrients and that the final size of a cabbage is inversely proportional to the number of other cabbages surrounding it.
   (a) A cabbage that had 12 other cabbages around it grew to 30 pounds. Find the constant of proportionality, and write the equation that expresses the inverse proportionality.
   (b) What size would this cabbage grow to if it had only 5 cabbage "neighbors"?

36. **Newton's Law of Gravitation** The gravitational force between two objects is inversely proportional to the square of the distance between them. How does the gravitational force between two objects change if the distance between them is (a) tripled? (b) halved?

37. **Heat of Campfire** The heat experienced by a hiker at a campfire is inversely proportional to the cube of his distance from the fire.
   (a) Write an equation that expresses the inverse proportionality.
   (b) If the hiker is too cold and moves halfway closer to the fire, how much more heat does the hiker experience?



38. **Frequency of Vibration** The frequency f of vibration of a violin string is inversely proportional to its length L. The constant of proportionality k is positive and depends on the tension and density of the string.
   (a) Write an equation that represents this proportionality.
   (b) What effect does doubling the length of the string have on the frequency of its vibration?