I6 quotions

7.3 Exercises

CONCEPTS

Fundamentals

1. Write the augmented matrix of the following system of equations.

System   **Augmented matrix**

$$\begin{cases} x + y - z = 1 \\ x \quad\quad + 2z = -3 \\ 2y - z = 3 \end{cases}$$

$$\begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$$

2. Write the system of equations that corresponds to the following augmented matrix.

Augmented matrix   **System**

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 3 & 0 & 1 & 2 \\ 5 & 2 & -1 & -2 \end{bmatrix}$$

$$\begin{cases} \underline{\hspace{3cm}} = \underline{\hspace{1cm}} \\ \underline{\hspace{3cm}} = \underline{\hspace{1cm}} \\ \underline{\hspace{3cm}} = \underline{\hspace{1cm}} \end{cases}$$

3. The matrix below is the augmented matrix of a system of linear equations in the variables x, y, and z. (It is given in reduced row-echelon form.)

$$\begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) The leading variables are _____ and _____.

(b) Is the system inconsistent or dependent? _____

(c) The solution of the system is

$$x = \underline{\hspace{1.5cm}} \quad\quad y = \underline{\hspace{1.5cm}} \quad\quad z = \underline{\hspace{1.5cm}}$$

4. The augmented matrix of a system of linear equations is given in reduced row-echelon form. Find the solution of the system.

(a) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

$x = \underline{\hspace{1cm}}$ $x = \underline{\hspace{1cm}}$ $x = \underline{\hspace{1cm}}$

$y = \underline{\hspace{1cm}}$ $y = \underline{\hspace{1cm}}$ $y = \underline{\hspace{1cm}}$

$z = \underline{\hspace{1cm}}$ $z = \underline{\hspace{1cm}}$ $z = \underline{\hspace{1cm}}$

SKILLS

5–10 ■ **(a)** State the dimensions of the matrix.
(b) Find the (2, 1) entry of the matrix.

5. $\begin{bmatrix} 2 & 7 \\ 0 & -1 \\ 5 & -3 \end{bmatrix}$ **6.** $\begin{bmatrix} -1 & 5 & 4 & 0 \\ 2 & 0 & 11 & 3 \end{bmatrix}$ **7.** $\begin{bmatrix} 12 \\ 35 \end{bmatrix}$

8. $\begin{bmatrix} -3 \\ 7 \\ 1 \end{bmatrix}$ **9.** $\begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$ **10.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

11–14   The system of linear equations has a unique solution. Find the solution using elementary row operations, as in Example 2.

11. $\begin{cases} x - 2y + z = 1 \\ \quad\; y + 2z = 5 \\ x + y + 3z = 8 \end{cases}$

12. $\begin{cases} x + y + 6z = 3 \\ x + y + 3z = 3 \\ x + 2y + 4z = 7 \end{cases}$

13. $\begin{cases} x + y + z = 2 \\ 2x - 3y + 2z = 4 \\ 4x + y - 3z = 1 \end{cases}$

14. $\begin{cases} x + y + z = 4 \\ -x + 2y + 3z = 17 \\ 2x - y = -7 \end{cases}$

15–20   ▧ Use a graphing calculator to put the augmented matrix of the system into row-echelon form. Then solve the system (as in Example 3).

15. $\begin{cases} x + 3y - 2z = -2 \\ x + 4y - 3z = -3 \\ x + 3y - z = 0 \end{cases}$

16. $\begin{cases} x + y - z = 4 \\ 2x + 3y - z = 5 \\ x + y = 1 \end{cases}$

17. $\begin{cases} x + z = 1 \\ 2x - y + 4z = -6 \\ x - y + 4z = -6 \end{cases}$

18. $\begin{cases} x - 3z = -5 \\ x + y - 2z = -4 \\ x + 2y = -1 \end{cases}$

19. $\begin{cases} x + 2y - z = -2 \\ x + z = 0 \\ 2x - y - z = -3 \end{cases}$

20. $\begin{cases} 2y + z = 4 \\ x + y = 4 \\ 3x + 3y - z = 10 \end{cases}$

21–24   ▧ Use a graphing calculator to put the augmented matrix of the system into reduced row-echelon form, then solve the system (as in Example 4).

21. $\begin{cases} x + 2y - z = 9 \\ 2x - z = -2 \\ 3x + 5y + 2z = 22 \end{cases}$

22. $\begin{cases} 2x + y = 7 \\ 2x - y + z = 6 \\ 3x - 2y + 4z = 11 \end{cases}$

23. $\begin{cases} 2x - 3y - z = 13 \\ -x + 2y - 5z = 6 \\ 5x - y - z = 49 \end{cases}$

24. $\begin{cases} 10x + 10y - 20z = 60 \\ 15x + 20y + 30z = -25 \\ -5x + 30y - 10z = 45 \end{cases}$

25–30   ▧ Use a graphing calculator to put the augmented matrix of the system into reduced row-echelon form. Determine whether the system is inconsistent or dependent. If it is dependent, find the complete solution.

25. $\begin{cases} x + y + z = 2 \\ y - 3z = 1 \\ 2x + y + 5z = 0 \end{cases}$

26. $\begin{cases} x + 3z = 3 \\ 2x + y - 2z = 5 \\ -y + 8z = 8 \end{cases}$

27. $\begin{cases} 2x - 3y - 9z = -5 \\ x + 3z = 2 \\ -3x + y - 4z = -3 \end{cases}$

28. $\begin{cases} x - 2y + 5z = 3 \\ -2x + 6y - 11z = 1 \\ 3x - 16y - 20z = -26 \end{cases}$

29. $\begin{cases} -2x + 6y - 2z = -12 \\ x - 3y + 2z = 10 \\ -x + 3y + 2z = 6 \end{cases}$

30. $\begin{cases} y - 5z = 7 \\ 3x + 2y = 12 \\ 3x + 10z = 80 \end{cases}$

31–40   ▧ Solve the system of linear equations.

31. $\begin{cases} 4x - 3y + z = -8 \\ -2x + y - 3z = -4 \\ x - y + 2z = 3 \end{cases}$

32. $\begin{cases} 2x - 3y + 5z = 14 \\ 4x - y - 2z = -17 \\ -x - y + z = 3 \end{cases}$

33. $\begin{cases} 2x + y + 3z = 9 \\ -x \quad\quad - 7z = 10 \\ 3x + 2y - z = 4 \end{cases}$

34. $\begin{cases} -4x - y + 36z = 24 \\ x - 2y + 9z = 3 \\ -2x + y + 6z = 6 \end{cases}$

35. $\begin{cases} x + 2y - 3z = -5 \\ -2x - 4y - 6z = 10 \\ 3x + 7y - 2z = -13 \end{cases}$

36. $\begin{cases} 3x + y \quad\quad = 2 \\ -4x + 3y + z = 4 \\ 2x + 5y + z = 0 \end{cases}$

37. $\begin{cases} x - y + 6z = 8 \\ x \quad\quad + z = 5 \\ x + 3y - 14z = -4 \end{cases}$

38. $\begin{cases} 3x - y + 2z = -1 \\ 4x - 2y + z = -7 \\ -x + 3y - 2z = -1 \end{cases}$

39. $\begin{cases} x - 3y + 2z + w = -2 \\ x - 2y \quad\quad - 2w = -10 \\ \quad\quad z + 5w = 15 \\ 3x \quad\quad + 2z + w = -3 \end{cases}$

40. $\begin{cases} x + y - z - w = 6 \\ 2x \quad\quad + z - 3w = 8 \\ x - y \quad\quad + 4w = -10 \\ 3x + 5y - z - w = 20 \end{cases}$

CONTEXTS

41. Nutrition A doctor recommends that a patient take 50 milligrams each of niacin, riboflavin, and thiamin daily to alleviate a vitamin deficiency. In his medicine chest at home the patient finds three brands of vitamin pills. The amounts of the relevant vitamins per pill are given in the table. How many pills of each type should he take every day to get 50 milligrams of each vitamin?

Vitamin (mg)	VitaMax	Vitron	VitaPlus
Niacin	5	10	15
Riboflavin	15	20	0
Thiamin	10	10	10

42. Mixtures A chemist has three acid solutions at various concentrations. The first is 10% acid, the second is 20%, and the third is 40%. How many milliliters of each should she use to make 100 milliliters of 18% solution if she has to use four times as much of the 10% solution as the 40% solution?

43. Distance, Speed, and Time Amanda, Bryce, and Corey enter a race in which they have to run, swim, and cycle over a marked course. Their average speeds are given in the table. Corey finishes first with a total time of 1 hour, 45 minutes. Amanda comes in second with a time of 2 hours, 30 minutes. Bryce finishes last with a time of 3 hours. Find the distance in miles for each part of the race.

Contestant	Average speed (mi/h)		
	Running	Swimming	Cycling
Amanda	10	4	20
Bryce	$7\frac{1}{2}$	6	15
Corey	15	3	40

44. Classroom Use A small school has 100 students who occupy three classrooms: A, B, and C. After the first period of the school day, half the students in room A move to room B, one-fifth of the students in room B move to room C, and one-third of the students in room C move to room A. Nevertheless, the total number of students in each room is the same for both periods. How many students occupy each room?

example 5
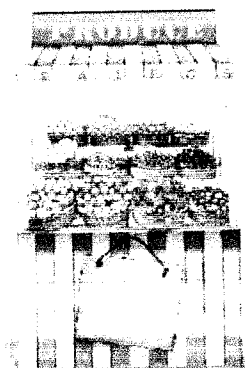
An organic farmer sells produce at an open-air market three days a week: Thursdays, Fridays, and Saturdays. He sell oranges, broccoli, and beans. Matrix A tabulates the number of pounds of produce he sold in a certain week. Matrix C gives the prices per pound (in dollars) he charged that week.

(a) Find the product AC.

(b) What was the total revenue on Friday?

(c) What was the total revenue for the week?

$$
A = \begin{array}{c}\text{Produce} \\ \begin{array}{ccc}\text{Oranges} & \text{Broccoli} & \text{Beans}\end{array} \\ \begin{bmatrix} 50 & 20 & 10 \\ 70 & 35 & 30 \\ 45 & 15 & 25 \end{bmatrix}\begin{array}{l}\leftarrow \text{Friday} \quad \text{Day}\\ \leftarrow \text{Saturday}\end{array}\end{array}
\qquad
C = \begin{bmatrix} 0.90 \\ 1.20 \\ 1.50 \end{bmatrix}\begin{array}{l}\leftarrow \text{Broccoli} \quad \text{Produce}\\ \leftarrow \text{Beans}\end{array}
$$

Solution

(a) The entries in the product matrix are obtained as explained above.

$$
AC = \begin{bmatrix} 50 & 20 & 10 \\ 70 & 35 & 30 \\ 45 & 15 & 25 \end{bmatrix}\begin{bmatrix} 0.90 \\ 1.20 \\ 1.50 \end{bmatrix} = \begin{bmatrix} 50\cdot0.90 + 20\cdot1.20 + 10\cdot1.50 \\ 70\cdot0.90 + 35\cdot1.20 + 30\cdot1.50 \\ 45\cdot0.90 + 15\cdot1.20 + 25\cdot1.50 \end{bmatrix} = \begin{bmatrix} 84.00 \\ 150.00 \\ 96.00 \end{bmatrix}
$$

(b) The (2, 1) entry of matrix AC is calculated as follows:

Pounds of oranges	Pounds of broccoli	Pounds of beans
Price per pound	Price per pound	Price per pound

$$70 \times 0.90 \; + \; 35 \times 1.20 \; + \; 30 \times 1.50 \; = \; 150.00$$

This means that the (2, 1) entry of matrix AC is the sum of Friday's revenue from oranges, broccoli, and beans. So the total revenue on Friday was $150.00.

(c) We add the three entries in the column matrix AC to get the total revenue for the week.

$$84.00 + 150.00 + 96.00 = 330.00$$

So the total revenue from all three days was $330.00.

■   **NOW TRY EXERCISE 17**   ■

7.4 Exercises

CONCEPTS

Fundamentals

1–4 Determine whether the data described is numerical or categorical data.

1. The height of each child in a third grade class.

2. The number of laundry items tabulated by color (white, pastel, dark).

3. The number of gulls seen at a beach on a given day, tabulated by species (California, Western, Ring-billed) and maturity (immature, juvenile, adult).

4. The number of birds observed at a bird feeder each day.

5–6 Organize the given set of categorical data into a data matrix.

5. Education and Income A women's club takes a survey to determine the education and income of its members. Of the women with no postsecondary education, 5 have a yearly income below $50,000, 2 have an income between $50,000 and $100,000, and 0 has an income above $100,000. Of the women with up to four years of postsecondary education, 2 have a yearly income below $50,000, 10 have an income between $50,000 and $100,000, and 4 have an income above $100,000. Of the women with more than four years of postsecondary education, 0 has a yearly income below $50,000, 3 have an income between $50,000 and $100,000, and 2 have an income above $100,000.

Postsecondary education (years)

None   0–4   More than 4

$$
A = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \leftarrow \text{More than } \$100,000
$$

6. Exam Scores A physics class takes a survey of the number of hours the students slept before the exam and the exam score. Of the students who slept less than four hours, six students got a score below 60, three got a score between 60 and 80, and one got a score above 80. Of the students who slept between four and seven hours, two students got a score below 60, seven got a score between 60 and 80, and four got a score above 80. Of the students who slept more than seven hours, three students got a score below 60, nine got a score between 60 and 80, and six got a score above 80.

Hours of sleep

Less than 4   4–7   More than 7

$$
A = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \leftarrow \text{Above 80} \quad \text{score}
$$

7. Service Ratings A Cincinnati car dealership conducted a customer satisfaction survey, and the results are tabulated in the data matrix A.

(a) How many of the customers that had a bill of less than $50 gave a fair service rating?

(b) How many customers had a bill above $100?

(c) How many customers gave an excellent service rating?

(d) How many customers were surveyed?

Amount of bill

$$
A = \begin{bmatrix} 10 & 8 & 5 \\ 5 & 7 & 6 \\ 1 & 3 & 5 \end{bmatrix} \begin{array}{l} \leftarrow \text{Excellent} \\ \leftarrow \text{Good} \\ \leftarrow \text{Fair} \end{array} \quad \text{Service rating}
$$

8. Service Ratings An Akron car dealership conducted a customer satisfaction survey, and the results are tabulated in the data matrix B.

(a) How many of the customers who had a bill of more than $100 gave an excellent service rating?

(b) How many customers had a bill less than $50?

(c) How many customers gave a good service rating?

(d) How many customers were surveyed?

$$B = \begin{bmatrix} 9 & 15 & 12 \\ 8 & 10 & 6 \\ 5 & 2 & 7 \end{bmatrix} \begin{array}{l} \leftarrow \text{Excellent} \\ \leftarrow \text{Good} \\ \end{array} \text{Service}$$

with column headers $\$50$, $\$50-100$, $\$100$

9. Health Insurance Residents of the small town of Springfield were surveyed to obtain information on the type of health insurance and the number of emergency room visits within the past year. The data are tabulated in matrix C.

(a) How many of those surveyed are members of an HMO and had four or more emergency visits in the past year?

(b) How many of those surveyed had no health insurance?

(c) How many of those surveyed had four or more emergency visits in the past year?

(d) How many people were surveyed?

Type of insurance

$$C = \begin{bmatrix} 11 & 8 & 3 \\ 5 & 6 & 5 \\ 3 & 2 & 7 \end{bmatrix} \leftarrow \text{4 or more}$$

with column headers HMO, Non-HMO, None

10. Health Insurance Residents of a Chicago suburb were surveyed to obtain information on the type of health insurance and the number of emergency room visits within the past year. The data are tabulated in matrix D.

(a) How many of those surveyed are members of an HMO and had four or more emergency visits in the past year?

(b) How many of those surveyed had no health insurance?

(c) How many of those surveyed had fewer than two emergency visits in the past year?

(d) How many people were surveyed?

$$D = \begin{bmatrix} 76 & 82 & 54 \\ 32 & 49 & 79 \\ 21 & 32 & 87 \end{bmatrix} \begin{array}{l} \leftarrow 0-1 \\ \leftarrow 2-3 \\ \leftarrow 4 \text{ or more} \end{array} \begin{array}{l} \text{Emergency} \\ \text{visits} \end{array}$$

11. Sales A tasting booth at Joe's Specialty Foods offers pesto pizza on Monday, spinach ravioli on Tuesday, and macaroni and cheese on Wednesday. The sales distribution for these products is tabulated in matrix A.

(a) Express each entry in the matrix as a proportion of the column total. Call the resulting matrix P.

(b) What does the $(1, 3)$ entry in P represent? What about the $(3, 3)$ entry?

$$A = \begin{bmatrix} 50 & 20 & 15 \\ 40 & 75 & 20 \\ 35 & 60 & 100 \end{bmatrix} \begin{array}{l} \leftarrow \text{Monday} \\ \leftarrow \text{Tuesday} \\ \leftarrow \text{Wednesday} \end{array} \text{Day}$$

12. Politics   A history class makes a survey of the students in the class, and the results are tabulated in matrix B.

(a) Express each entry in the matrix as a proportion of the column total. Call the resulting matrix P.

(b) What does the (2, 2) entry in P represent? What about the (1, 3) entry?

$$B = \begin{bmatrix} 7 & 9 & 3 \\ 8 & 10 & 2 \end{bmatrix} \begin{matrix} \leftarrow \text{Male} \\ \leftarrow \text{Female} \end{matrix} \text{Gender}$$

13. Service Ratings   The following questions refer to matrices A and B from Exercises 7 and 8.

(a) Find a data matrix for the results of the survey in Exercise 7 combined with the results of the survey in Exercise 8.

(b) What is the total number of customers (from both surveys) who had a bill between $50 and $100 and gave an excellent service rating?

14. Health Insurance   The following questions refer to matrices C and D from Exercises 9 and 10.

(a) Find a data matrix for the results of the survey in Exercise 9 combined with the results of the survey in Exercise 10.

(b) What is the total number of those surveyed (from both surveys) that had no health insurance and had four or more emergency visits in the past year?

15. Sales   The following questions refer to matrix A in Exercise 11.

(a) Find the scalar product $4A$.

(b) In this new data matrix, what proportion of the pesto pizza sales occurs on Wednesday?

16. Politics   The following questions refer to matrix B in Exercise 12.

(a) Find the scalar product $5B$.

(b) In this new data matrix, what proportion of the female students are Republican?

17. Car-Manufacturing Profits   A specialty car manufacturer has plants in Auburn, Biloxi, and Chattanooga. Three models are produced, with daily production given in matrix A. The profit (in dollars) per car is tabulated by model in matrix C.

(a) Find the product matrix AC.

(b) Assuming that all cars produced are sold, what is the daily profit from the Biloxi plant?

(c) What is the total daily profit (from all three plants)?

Cars produced each day

$$A = \begin{matrix} \text{Model K} & \text{Model R} & \text{Model W} \\ \downarrow & \downarrow & \downarrow \end{matrix} \\ \begin{bmatrix} 12 & 10 & 0 \\ 4 & 4 & 20 \\ 8 & 9 & 12 \end{bmatrix} \begin{matrix} \leftarrow \text{Auburn} \\ \\ \end{matrix}$$

Profit

$$C = \begin{bmatrix} 1000 \\ 2000 \\ 1500 \end{bmatrix} \begin{matrix} \leftarrow \text{Model K} \\ \leftarrow \text{Model R} \\ \end{matrix}$$