

Experiment, Outcome, and Sample Space

There are only a few things, if any, in life that have definite, certain, and sure outcomes. Most things in life have uncertain outcomes. For example, will you get an A grade in the statistics class that you are taking this semester? Suppose you recently opened a new business; will it be successful? Your friend will be getting married next month. Will she be happily married for the rest of her life? You just bought a lottery ticket. Will it be a winning ticket? You are playing black jack at a casino. Will you be a winner after 20 plays of the game? In all these situations, the outcomes are random. You know the various outcomes for each of these games/statistical experiments, but you do not know which one of these outcomes will actually occur. As another example, suppose you are working as a quality control manager at a factory where they are making hockey pucks. You randomly select a few pucks from the production line and inspect them for being good or defective. This act of inspecting a puck is an example of a statistical **experiment**. The result of this inspection will be that the puck is either “good” or “defective.” Each of these two observations is called an **outcome** (also called a **basic** or **final outcome**) of the experiment, and these outcomes taken together constitute the **sample space** for this experiment. The elements of a sample space are called **sample points**.

Experiment, Outcomes, and Sample Space An **experiment** is a process that, when performed, results in one and only one of many observations. These observations are called the **outcomes** of the experiment. The collection of all outcomes for an experiment is called a **sample space**.

Table 4.1 lists some examples of experiments, their outcomes, and their sample spaces.

Table 4.1 Examples of Experiments, Outcomes, and Sample Spaces

| Experiment | Outcomes | Sample Space |
|-------------------|------------------|--------------------|
| Toss a coin once | Head, Tail | {Head, Tail} |
| Roll a die once | 1, 2, 3, 4, 5, 6 | {1, 2, 3, 4, 5, 6} |
| Toss a coin twice | HH, HT, TH, TT | {HH, HT, TH, TT} |
| Play lottery | Win, Lose | {Win, Lose} |
| Take a test | Pass, Fail | {Pass, Fail} |
| Select a worker | Male, Female | {Male, Female} |

The sample space for an experiment can also be illustrated by drawing a tree diagram. In a **tree diagram**, each outcome is represented by a branch of the tree. Tree diagrams help us understand probability concepts by presenting them visually. Examples 4-1 through 4-3 describe how to draw the tree diagrams for statistical experiments.

Draw the tree diagram for the experiment of tossing a coin once.

EXAMPLE 4-1 One Toss of a Coin

Draw the tree diagram for the experiment of tossing a coin once.

Solution This experiment has two possible outcomes: head and tail. Consequently, the sample space is given by

Sample space = $\{H, T\}$, where H = Head and T = Tail

To draw a tree diagram, we draw two branches starting at the same point, one representing the head and the second representing the tail. The two final outcomes are listed at the ends of the branches as shown in Figure 4.1.

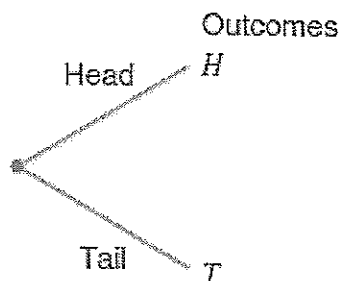


Figure 4.1 Tree diagram for one toss of a coin.

Drawing the tree diagram: two tosses of a coin.

EXAMPLE 4-2 Two Tosses of a Coin

Draw the tree diagram for the experiment of tossing a coin twice.

Solution This experiment can be split into two parts: the first toss and the second toss. Suppose that the first time the coin is tossed, we obtain a head. Then, on the second toss, we can still obtain a head or a tail. This gives us two outcomes: HH (head on both tosses) and HT (head on the first toss and tail on the second toss). Now suppose that we observe a tail on the first toss. Again, either a head or a tail can occur on the second toss, giving the remaining two outcomes: TH (tail on the first toss and head on the second toss) and TT (tail on both tosses). Thus, the sample space for two tosses of a coin is

$$\text{Sample space} = \{HH, HT, TH, TT\}$$

The tree diagram is shown in Figure 4.2. This diagram shows the sample space for this experiment.

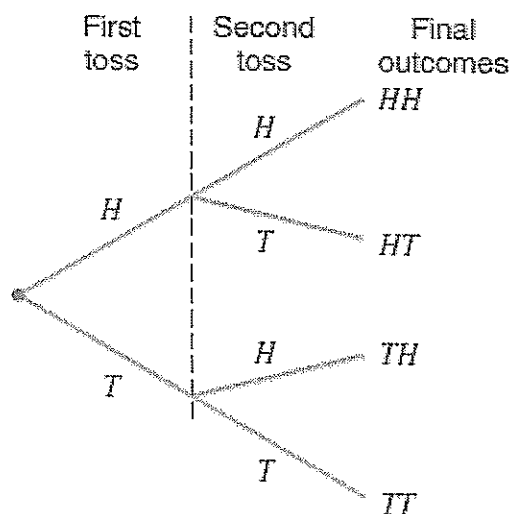


Figure 4.2 Tree diagram for two tosses of a coin.

Drawing the tree diagram: two selections.

EXAMPLE 4-3 Selecting Two Workers

Suppose we randomly select two workers from a company and observe whether the worker selected each time is a man or a woman. Write all the outcomes for this experiment. Draw the tree diagram for this experiment.

Solution Let us denote the selection of a man by M and that of a woman by W. We can compare the selection of two workers to two tosses of a coin. Just as each toss of a coin can result in one of two outcomes, head or tail, each selection from the workers of this company can result in one of two outcomes, man or woman. As we can see from the tree diagram of Figure 4.3, there are four final outcomes: MM, MW, WM, WW. Hence, the sample space is written as

Sample space = {MM, MW, WM, WW}

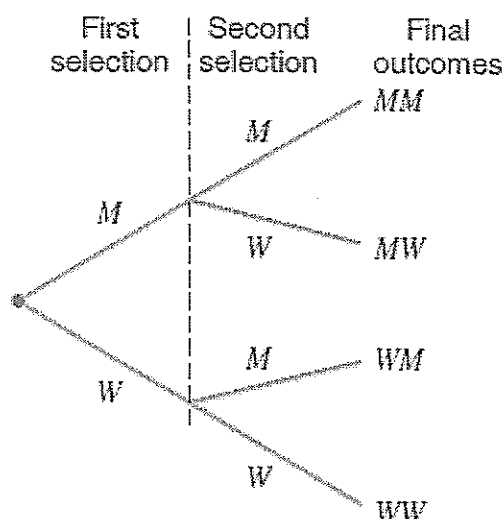


Figure 4.3 Tree diagram for selecting two workers.

4.1.1 Simple and Compound Events

An **event** consists of one or more of the outcomes of an experiment.

Event An **event** is a collection of one or more of the outcomes of an experiment.

An event may be a **simple event** or a **compound event**. A simple event is also called an **elementary event**, and a compound event is also called a **composite event**.

Simple Event

Each of the final outcomes for an experiment is called a **simple event**. In other words, a simple event includes one and only one outcome. Usually, simple events are denoted by E_1 , E_2 , E_3 , and so forth. However, we can denote them by any other letter—that is, by A , B , C , and so forth. Many times we denote events by the same letter and use subscripts to distinguish them, as in A_1, A_2, A_3, \dots .

Simple Event An event that includes one and only one of the (final) outcomes for an experiment is called a **simple event** and is usually denoted by E_i .

Example 4-4 describes simple events.

Illustrating simple events.

EXAMPLE 4-4 Selecting Two Workers

Reconsider Example 4-3 on selecting two workers from a company and observing whether the worker selected each time is a man or a woman. Each of the final four outcomes (MM, MW, WM, and WW) for this experiment is a simple event. These four events can be denoted by E_1 , E_2 , E_3 , and E_4 , respectively. Thus,

$$E_1 = \{MM\}, \quad E_2 = \{MW\}, \quad E_3 = \{WM\}, \quad \text{and} \quad E_4 = \{WW\}$$

Compound Event

A **compound event** consists of more than one outcome.

Compound Event A **compound event** is a collection of more than one outcome for an experiment.

Compound events are denoted by A, B, C, D... , or by A_1, A_2, A_3, \dots , or by B_1, B_2, B_3, \dots , and so forth. Examples 4-5 and 4-6 describe compound events.

Illustrating a compound event: two selections.

EXAMPLE 4-5 Selecting Two Workers

Reconsider Example 4-5 on selecting two workers from a company and observing whether the worker selected each time is a man or a woman. Let A be the event that at most one man is selected. Is event A a simple or a compound event?

Solution Here *at most one man* means one or no man is selected. Thus, event A will occur if either no man or one man is selected. Hence, the event A is given by

$$A = \text{at most one man is selected} = \{MW, WM, WW\}$$

Because event A contains more than one outcome, it is a compound event. The diagram in Figure 4.4 gives a graphic presentation of compound event A.

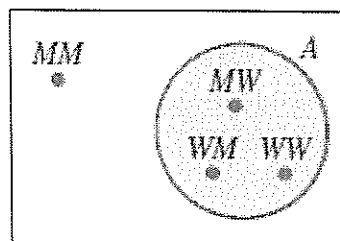


Figure 4.4 Diagram showing event A.

Illustrating simple and compound events: two selections.

EXAMPLE 4-6 Preference for Ice Tea

In a group of college students, some like ice tea and others do not. There is no student in this group who is indifferent or has no opinion. Two students are randomly selected from this group.

- a. How many outcomes are possible? List all the possible outcomes.
- b. Consider the following events. List all the outcomes included in each of these events. Mention whether each of these events is a simple or a compound event.
 - i. Both students like ice tea.
 - ii. At most one student likes ice tea.
 - iii. At least one student likes ice tea.
 - iv. Neither student likes ice tea.

Solution Let L denote the event that a student likes ice tea and N denote the event that a student does not like ice tea.

- a. This experiment has four outcomes, which are listed below and shown in Figure 4.5.

LL = Both students like ice tea
 LN = The first student likes ice tea but the second student does not
 NL = The first student does not like ice tea but the second student does
 NN = Both students do not like ice tea

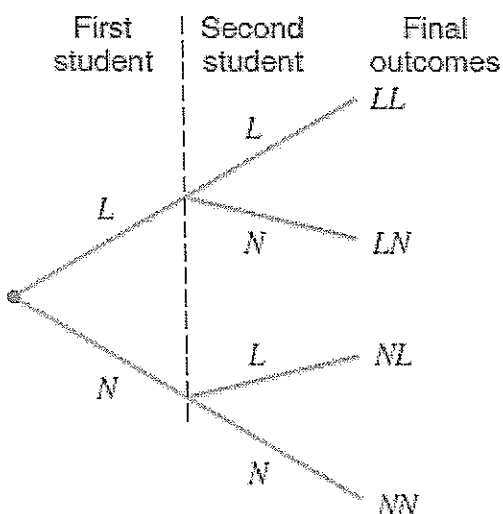


Figure 4.5 Tree diagram.

- b.
 - i. The event *both students like ice tea* will occur if LL happens. Thus,

$$\text{Both students like ice tea} = \{LL\}$$

Since this event includes only one of the four outcomes, it is a **simple** event.

ii.

The event *at most one student likes ice tea* will occur if one or none of the two students likes ice tea, which will include the events LN , NL , and NN . Thus,

$$\text{At most one student likes ice tea} = \{LN, NL, NN\}$$

Since this event includes three outcomes, it is a **compound** event.

- iii. The event *at least one student likes ice tea* will occur if one or two of the two students like ice tea, which will include the events LN , NL , and LL . Thus,

$$\text{At least one student likes ice tea} = \{LN, NL, LL\}$$

Since this event includes three outcomes, it is a **compound** event.

- iv. The event *neither student likes ice tea* will occur if neither of the two students likes ice tea, which will include the event NN . Thus,

$$\text{Neither student likes ice tea} = \{NN\}$$

Since this event includes one outcome, it is a **simple** event.

EXERCISES

CONCEPTS AND PROCEDURES

4.1 Define the following terms: *experiment*, *outcome*, *sample space*, *simple event*, and *compound event*.

4.2 List the simple events for each of the following statistical experiments in a sample space.

- One roll of a die
- Three tosses of a coin
- One toss of a coin and one roll of a die

4.3 A box contains three items that are labeled A, B, and C. Two items are selected at random (without replacement) from this box. List all the possible outcomes for this experiment. Write the sample space.

APPLICATIONS

4.4 Two students are randomly selected from a statistics class, and it is observed whether or not they suffer from math anxiety. How many total outcomes are possible? Draw a tree diagram for this experiment.

4.5 In a group of adults, some own iPads, and others do not. If two adults are randomly selected from this group, how many total outcomes are possible? Draw a tree diagram for this experiment.

4.6 An automated teller machine at a local bank is stocked with \$10 and \$20 bills. When a customer withdraws \$40 from the machine, it dispenses either two \$20 bills or four \$10 bills. If two customers withdraw \$40 each, how many outcomes are possible? Draw a tree diagram for this experiment.

4.7 In a group of people, some are in favor of a tax increase on rich people to reduce the federal deficit and others are against it. (Assume that there is no other outcome such as “no opinion” and “do not know.”) Three persons are selected at random from this group and their opinions in favor or against raising such taxes are noted. How many total outcomes are possible? Write these outcomes in a sample space. Draw a tree diagram for this experiment.

4.8 Two students are randomly selected from a statistics class, and it is observed whether or not they suffer from math anxiety. List all the outcomes included in each of the following events. Indicate which are simple and which are compound events.

- Both students suffer from math anxiety.
- Exactly one student suffers from math anxiety.
- The first student does not suffer and the second suffers from math anxiety.
- None of the students suffers from math anxiety.

4.9 In a group of adults, some own iPads, and others do not. Two adults are randomly selected from this group. List all the outcomes included in each of the following events. Indicate which are simple and which are compound events.

- One person owns an iPad and the other does not.
- At least one person owns an iPad.
- Not more than one person owns an iPad.
- The first person owns an iPad and the second does not.

4.10 An automated teller machine at a local bank is stocked with \$10 and \$20 bills. When a customer withdraws \$40 from this machine, it dispenses either two \$20 bills or four \$10 bills. Two customers withdraw \$40 each. List all of the outcomes in each of the following events and mention which of these are simple and which are compound events.

- Exactly one customer receives \$20 bills.
- Both customers receive \$10 bills.
- At most one customer receives \$20 bills.
- The first customer receives \$10 bills and the second receives \$20 bills.

4.2 Calculating Probability

probability, which gives the likelihood of occurrence of an event, is denoted by P . The probability that a simple event E_i will occur is denoted by $P(E_i)$, and the probability that a compound event A will occur is denoted by $P(A)$.

Probability **Probability** is a numerical measure of the likelihood that a specific event will occur.

4.2.1 Two Properties of Probability

There are two important properties of probability that we should always remember. These properties are mentioned below.

1. The probability of an event always lies in the range 0 to 1.

Whether it is a simple or a compound event, the probability of an event is never less than 0 or greater than 1. We can write this property as follows.

First Property of Probability

$$0 \leq P(E_i) \leq 1$$

$$0 \leq P(A) \leq 1$$

An event that cannot occur has zero probability and is called an **impossible event**. An event that is certain to occur has a probability equal to 1 and is called a **sure event**. In the following examples, the first event is an impossible event and the second one is a sure event.

$$P(\text{a tossed coin will stand on its edge}) = 0$$

$$P(\text{a child born today will eventually die}) = 1.0$$

There are very few events in real life that have probability equal to either zero or 1.0. Most of the events in real life have probabilities that are between zero and 1.0. In other words, these probabilities are greater than zero but less than 1.0. A higher probability such as .82 indicates that the event is more likely to occur. On the other hand, an event with a lower probability such as .12 is less likely to occur. Sometime events with very low (.05 or lower) probabilities are also called **rare events**.

2. The sum of the probabilities of all simple events (or final outcomes) for an experiment, denoted by $\sum P(E_i)$, is always 1.

Second Property of Probability For an experiment with outcomes E_1, E_2, E_3, \dots ,

$$\sum P(E_i) = P(E_1) + P(E_2) + P(E_3) + \dots = 1.0$$

For example, if you buy a lottery ticket, you may either win or lose. The probabilities of these two events must add to 1.0, that is:

$$P(\text{you will win}) + P(\text{you will lose}) = 1.0$$

Similarly, for the experiment of one toss of a coin,

$$P(\text{Head}) + P(\text{Tail}) = 1.0$$

For the experiment of two tosses of a coin,

$$P(\text{HH}) + P(\text{HT}) + P(\text{TH}) + P(\text{TT}) = 1.0$$

For one game of football by a professional team,

$$P(\text{win}) + P(\text{loss}) + P(\text{tie}) = 1.0$$

4.2.2 Three Conceptual Approaches to Probability

How do we assign probabilities to events? For example, we may say that the probability of obtaining a head in one toss of a coin is .50, or that the probability that a randomly selected family owns a home is .68, or that the Los Angeles Dodgers will win the Major League Baseball championship next year is .14. How do we obtain these probabilities? We will learn the procedures that are used to obtain such probabilities in this section. There are three conceptual approaches to probability: (1) classical probability, (2) the relative frequency concept of probability, and (3) the subjective probability concept. These three concepts are explained next.

Classical Probability

Many times, various outcomes for an experiment may have the same probability of occurrence. Such outcomes are called **equally likely outcomes**. The classical probability rule is applied to compute the probabilities of events for an experiment for which all outcomes are equally likely. For example, head and tail are two equally likely outcomes when a fair coin is tossed once. Each of these two outcomes has the same chance of occurrence.

Equally Likely Outcomes Two or more outcomes that have the same probability of occurrence are said to be **equally likely outcomes**.

Earlier in this section, we learned that the total probability for all simple outcomes of an experiment is 1.0. This total probability is distributed over all the outcomes of the experiment. When all the outcomes of an experiment are equally likely, this total probability will be equally distributed over various outcomes. For example, for a fair coin, there are two equally likely outcomes—a head and a tail. Thus, if we distribute the total probability of 1.0 equally among these two outcomes, then each of these outcomes will have a probability of .50 of occurrence.

According to the **classical probability rule**, to find the probability of a simple event, we divide 1.0 by the total number of outcomes for the experiment. On the other hand, to find the probability of a compound event A , we divide the number of outcomes favorable to event A by the total number of outcomes for the experiment.

Classical Probability Rule to Find Probability Suppose E_i is a simple event and A is a compound event for an experiment with equally likely outcomes. Then, applying the classical approach, the probabilities of E_i and A are:

$$P(E_i) = \frac{1}{\text{Total number of outcomes for the experiment}}$$

$$P(A) = \frac{\text{Number of outcomes favorable to } A}{\text{Total number of outcomes for the experiment}}$$

Examples 4-7 through 4-9 illustrate how probabilities of events are calculated using the classical probability rule.

Calculating the probability of a simple event.

EXAMPLE 4-7 One Toss of a Coin

Find the probability of obtaining a head and the probability of obtaining a tail for one toss of a coin.

Solution If we toss a fair coin once, the two outcomes, head and tail, are equally likely outcomes. Hence, this is an example of a classical experiment. Therefore,

$$P(\text{head}) = \frac{1}{\text{Total number of outcomes}} = \frac{1}{2} = .50$$

Similarly,

$$P(\text{tail}) = \frac{1}{2} = .50$$

Since this experiment has only two outcomes, a head and a tail, their probabilities add to 1.0.

Calculating the probability of a compound event.

EXAMPLE 4-8 One Roll of a Die

Find the probability of obtaining an even number in one roll of a die.

Solution This experiment of rolling a die once has a total of six outcomes: 1, 2, 3, 4, 5, and 6. Given that the die is fair, these outcomes are equally likely. Let A be an event that an even number is observed on the die. Event A includes three outcomes: 2, 4, and 6; that is,

$$A = \text{an even number is obtained} = \{2, 4, 6\}$$

If any one of these three numbers is obtained, event A is said to occur. Since three out of six outcomes are included in the event that an even number is obtained, its probability is:

$$P(A) = \frac{\text{Number of outcomes included in A}}{\text{Total number of outcomes}} = \frac{3}{6} = .50$$

Calculating the probability of a compound event.

EXAMPLE 4-9 Selecting One Out of Five Homes

Jim and Kim have been looking for a house to buy in New Jersey. They like five of the homes they have looked at recently and two of those are in West Orange. They cannot decide which of the five homes they should pick to make an offer. They put five balls (of the same size) marked 1 through 5 (each number representing a home) in a box and asked their daughter to select one of these balls. Assuming their daughter's selection is random, what is the probability that the selected home is in West Orange?

Solution With random selection, each home has the same probability of being selected and the five outcomes (one for each home) are equally likely. Two of the five homes are in West Orange. Hence,

$$P(\text{selected home is in West Orange}) = \frac{2}{5} = .40$$

Relative Frequency Concept of Probability

Suppose we want to calculate the following probabilities:

1. The probability that the next car that comes out of an auto factory is a "lemon"
2. The probability that a randomly selected family owns a home

3. The probability that a randomly selected woman is an excellent driver
4. The probability that an 80-year-old person will live for at least 1 more year
5. The probability that a randomly selected adult is in favor of increasing taxes to reduce the national debt
6. The probability that a randomly selected person owns a sport-utility vehicle (SUV)

These probabilities cannot be computed using the classical probability rule because the various outcomes for the corresponding experiments are not equally likely. For example, the next car manufactured at an auto factory may or may not be a lemon. The two outcomes, “the car is a lemon” and “the car is not a lemon,” are not equally likely. If they were, then (approximately) half the cars manufactured by this company would be lemons, and this might prove disastrous to the survival of the factory.

Although the various outcomes for each of these experiments are not equally likely, each of these experiments can be performed again and again to generate data. In such cases, to calculate probabilities, we either use past data or generate new data by performing the experiment a large number of times. Using these data, we calculate the frequencies and relative frequencies for various outcomes. The relative frequency of an event is used as an approximation for the probability of that event. This method of assigning a probability to an event is called the **relative frequency as an approximation of probability**. Because relative frequencies are determined by performing an experiment, the probabilities calculated using relative frequencies may change when an experiment is repeated. For example, every time a new sample of 500 cars is selected from the production line of an auto factory, the number of lemons in those 500 cars is expected to be different. However, the variation in the percentage of lemons will be small if the sample size is large. Note that if we are considering the population, the relative frequency will give an exact probability.

Using Relative Frequency as an Approximation of Probability If an experiment is repeated n times and an event A is observed f times where f is the frequency, then, according to the relative frequency concept of probability:

$$P(A) = \frac{f}{n} = \frac{\text{Frequency of } A}{\text{Sample size}}$$

Examples 4-10 and 4-11 illustrate how the probabilities of events are approximated using the relative frequencies.

Approximating probability by relative frequency: sample data.

EXAMPLE 4-10 Lemons in Car Production

Ten of the 500 randomly selected cars manufactured at a certain auto factory are found to be lemons. Assuming that lemons are manufactured randomly, what is the probability that the next car manufactured at this auto factory is a lemon?

Solution Let n denote the total number of cars in the sample and f the number of lemons in n . Then, from the given information:

$$n = 500 \quad \text{and} \quad f = 10$$

Using the relative frequency concept of probability, we obtain

$$P(\text{next car is a lemon}) = \frac{f}{n} = \frac{10}{500} = .02$$

This probability is actually the relative frequency of lemons in 500 cars. Table 4.2 lists the frequency and relative frequency distributions for this example.

Table 4.2 Frequency and Relative Frequency Distributions for the Sample of Cars

| Car | f | Relative Frequency |
|-------|-----------|--------------------|
| Good | 490 | $490/500 = .98$ |
| Lemon | 10 | $10/500 = .02$ |
| | $n = 500$ | Sum = 1.00 |

The column of relative frequencies in Table 4.2 is used as the column of approximate probabilities. Thus, from the relative frequency column,

$$P(\text{next car is a lemon}) = .02$$

$$P(\text{next car is a good car}) = .98$$

Law of Large Numbers

Note that relative frequencies are not exact probabilities but are approximate probabilities unless they are based on a census. However, if the experiment is repeated again and again, this approximate probability of an outcome obtained from the relative frequency will approach the actual probability of that outcome. This is called the **law of large numbers**.

Law of Large Numbers If an experiment is repeated again and again, the probability of an event obtained from the relative frequency approaches the actual (or theoretical) probability.

We used Minitab to simulate the tossing of a coin with a different number of tosses. Table 4.3 lists the results of these simulations.

Table 4.3 Simulating the Tosses of a Coin

| Number of Tosses | Number of Heads | Number of Tails | $P(H)$ | $P(T)$ |
|------------------|-----------------|-----------------|--------|--------|
| 3 | 3 | 3 | .00 | 1.00 |
| 8 | 6 | 2 | .75 | .25 |
| 25 | 9 | 16 | .36 | .64 |
| 100 | 61 | 39 | .61 | .39 |
| 1000 | 522 | 478 | .522 | .478 |
| 10,000 | 4962 | 5038 | .4962 | .5038 |
| 1,000,000 | 500,313 | 499,687 | .5003 | .4997 |

As we can observe from this table, when the coin was tossed three times, based on relative frequencies, the probability of a head obtained from this simulation was .00 and that of a tail was 1.00. When the coin was tossed eight times, the probability of a head obtained from this simulation was .75 and that of a tail was .25. Notice how these probabilities change. As the number of tosses increases, the probabilities of both a head and a tail converge toward .50. When the simulated tosses of the coin are 1,000,000, the relative frequencies of head and tail are almost equal. This is what is meant by the **Law of Large Numbers**. If the experiment is repeated only a few times, the probabilities obtained may not be close to the actual probabilities. As the number of repetitions increases, the probabilities of outcomes obtained become very close to the actual probabilities. Note that in the example of tossing a fair coin, the actual probability of head and tail is .50 each.

Approximating probability by relative frequency.

EXAMPLE 4-11 Owning a Home

Allison wants to determine the probability that a randomly selected family from New York State owns a home. How can she determine this probability?

Solution There are two outcomes for a randomly selected family from New York State: “This family owns a home” and “This family does not own a home.” These two outcomes are not equally likely. (Note that these two outcomes will be equally likely if exactly half of the families in New York State own homes and exactly half do not own homes.) Hence, the classical probability rule cannot be applied. However, we can repeat this experiment again and again. In other words, we can select a sample of families from New York State and observe whether or not each of them owns a home. Hence, we will use the relative frequency approach to probability.

Suppose Allison selects a random sample of 1000 families from New York State and observes that 730 of them own homes and 270 do not own homes. Then,

$$n = \text{sample size} = 1000$$

$$f = \text{number of families who own homes} = 730$$

Consequently,

$$P(\text{a randomly selected family owns a home}) = \frac{f}{n} = \frac{730}{1000} = .730$$

Again, note that .730 is just an approximation of the probability that a randomly selected family from New York State owns a home. Every time Allison repeats this experiment she may obtain a different probability for this event. However, because the sample size ($n=1000$) in this example is large, the variation is expected to be relatively small.

Subjective Probability

Many times we face experiments that neither have equally likely outcomes nor can be repeated to generate data. In such cases, we cannot compute the probabilities of events using the classical probability rule or the relative frequency concept. For example, consider the following probabilities of events:

1. The probability that Carol, who is taking a statistics course, will earn an A in the course
2. The probability that the Dow Jones Industrial Average will be higher at the end of the next trading day
3. The probability that the New York Giants will win the Super Bowl next season
4. The probability that Joe will lose the lawsuit he has filed against his landlord

Neither the classical probability rule nor the relative frequency concept of probability can be applied to calculate probabilities for these examples. All these examples belong to experiments that have neither equally likely outcomes nor the potential of being repeated. For example, Carol, who is taking statistics, will take the test (or tests) only once, and based on that she will either earn an A or not. The two events “she will earn an A” and “she will not earn an A” are not equally likely. Also, she cannot take the test (or tests) again and again to calculate the relative frequency of getting or not getting an A grade. She will take the test (or tests) only once. The probability assigned to an event in such cases is called **subjective probability**. It is based on the individual's judgment, experience, information, and belief. Carol may be very confident and assign a higher probability to the event that she will earn an A in statistics, whereas her instructor may be more cautious and assign a lower probability to the same event.

Subjective Probability **Subjective probability** is the probability assigned to an event based on subjective judgment, experience, information, and belief. There are no definite rules to assign such probabilities.

Subjective probability is assigned arbitrarily. It is usually influenced by the biases, preferences, and experience of the person assigning the probability.

EXERCISES

CONCEPTS AND PROCEDURES

- 4.11 Briefly explain the two properties of probability.
- 4.12 Briefly describe an impossible event and a sure event. What is the probability of the occurrence of each of these two events?
- 4.13 Briefly explain the three approaches to probability. Give one example of each approach.

4.14 Briefly explain for what kind of experiments we use the classical approach to calculate probabilities of events and for what kind of experiments we use the relative frequency approach.

4.15 Which of the following values cannot be the probability of an event and why?

| | | | | | | | |
|-----|-------|--------|-----|-------|--------|-----|---------|
| 2.4 | $3/8$ | $-.63$ | .55 | $9/4$ | $-2/9$ | 1.0 | $12/17$ |
|-----|-------|--------|-----|-------|--------|-----|---------|

APPLICATIONS

4.16 An economist says that the probability is .47 that a randomly selected adult is in favor of keeping the Social Security system as it is, .32 that this adult is in favor of totally abolishing the Social Security system, and .21 that this adult does not have any opinion or is in favor of other options. Were these probabilities obtained using the classical approach, relative frequency approach, or the subjective probability approach? Explain your answer.

4.17 The president of a company has a hunch that there is a .80 probability that the company will be successful in marketing a new brand of ice cream. Is this a case of classical, relative frequency, or subjective probability? Explain why.

4.18