Helpful Example to Solve Question 1 and 2 for HW4 IJL

Example 6.5

A bare spherical thermal reactor, 100 cm in radius, consists of a homogeneous mixture of ^{235}U and graphite. The reactor is critical and operates at a power level of 100 thermal kilowatts. Using modified one-group theory, calculate (a) the buckling; (b) the critical mass; (c) k_∞; (d) L_T^2; (e) the thermal flux. For simplicity, make all computations at room temperature.

Solution.

1. From Table 6.2, $B_2 = (\pi/R)^2 = (\pi/100)^2 = 9.88 \times 10^{-4}$ cm^{-2}, and $B = 3.14 \times 10^{-2}$. [*Ans.*]
2. According to Tables 5.2, 5.3, and 6.3, $L_{TM}^2 = 3500$ cm^2, $\eta_T = 2.065$, and $\tau_{TM} = 368$ cm^2. Then from Eq. (6.83),

$$Z = \frac{1 + 9.88 \times 10^{-4}(3500 + 368)}{2.065 - 1 - 9.88 \times 10^{-4} \times 368} = 6.87.$$

Using the values $\sigma_{aM}(E_0) = 0.0034$ b, $\sigma_{aF}(T) = 681$ b, $g_a(T) = 0.978$ in Eq. (6.88) gives

$$m_F = \frac{6.87 \times 0.0034 \times 235}{0.978 \times 681 \times 12} m_M = 6.87 \times 10^{-4} m_M.$$

The density of graphite is approximately 1.60 g/cm^3, so the total mass of graphite in the reactor is $m_M = \frac{4}{3}\pi R^3 \times 1.60 = 6.70 \times 10^6$ g $= 6,700$ kg. It follows that the critical mass is $m_F = 6.87 \times 10^{-4} \times 6,700 = 4.60$ kg. [*Ans.*]

3. From Eq. (6.79),

$$f = \frac{Z}{Z+1} = \frac{6.87}{7.87} = 0.873,$$

and

$$k_\infty = \eta_T f = 2.065 \times 0.873 = 1.803. \text{ [\textit{Ans.}]}$$

4. From Eq. (6.82),

$$L_T^2 = (1-f)L_{TM}^2 = (1 - 0.873) \times 3500 = 444 \text{ cm.}^2 \text{ [\textit{Ans.}]}$$

5. The thermal flux is given by

$$\phi_T = A\frac{\sin Br}{r},$$

where, according to Table 6.2, $A = P/4R^2 E_R \Sigma_f$. Here $P = 100$ kW$= 10^5$ joules/sec and $E_R = 3.2 \times 10^{-11}$ joule. The value of $\phi\Sigma_f = N_F\phi\sigma_f$ can be found by noting from Eq. (6.85) that

$$N_F = \frac{m_F N_A}{V M_F},$$

so that

$$\phi\Sigma_f = \frac{m_F N_A \phi\sigma_f}{V M_F}$$
$$= \frac{m_F N_A}{V M_F} \times 0.886 g_{fF}(T)\sigma_f(E_0).$$

Using the values $g_{fF}(T) = 0.976$ and $\sigma_f(E_0) = 582$ b gives $\phi\Sigma_f = 1.41 \times 10^{-3}$ cm^{-1}. The constant A is then

$$A = \frac{10^5}{4 \times 10^4 \times 3.2 \times 10^{-11} \times 1.41 \times 10^{-3}} = 5.54 \times 10^{13}$$

and the flux is for d small

$$\phi_T(r) = 5.54 \times 10^{13}\frac{\sin Br}{r}. \text{ [\textit{Ans.}]}$$

The maximum value of ϕ_T occurs at $r = 0$ and is equal to $\phi_T(0) = 5.54 \times 10^{13} B = 1.74 \times 10^{12}$ neutrons/cm^2-sec.

Helpful to solve Question 3 for HW4 IJL

(2)

Example 6.12

The core of an experimental reactor consists of a square lattice of natural uranium rods imbedded in graphite. The rods are 1.02 cm in radius and 25.4 cm apart.[8] Calculate the value of f for this reactor.

Solution. It is first necessary to determine the radius b of the cylindrical cell having the same volume as the unit cell. Both cells are shown in Fig. 6.9. These cells are of equal length, so for equal volumes it is necessary that

$$\pi b^2 = (25.4)^2$$

and so

$$b = 25.4/\sqrt{\pi} = 14.3 \text{ cm}.$$

[8]The distance between the centers of nearest rods, 25.4 cm in the example, is called the *lattice spacing* or *lattice pitch*.

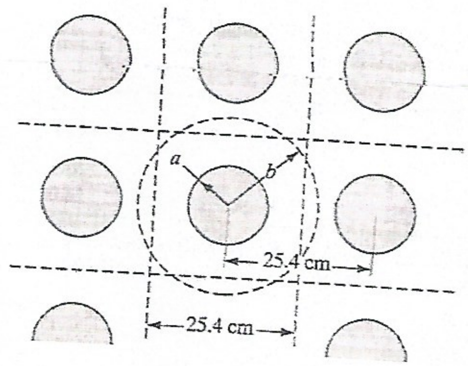

Figure 6.9 A square lattice with 25.4-cm pitch.

For natural uranium, $L_T = 1.55$ cm; for graphite, $L_T = 59$ cm. Thus,

$$x = 1.02/1.55 = 0.658,$$
$$y = 1.02/59 = 0.0173,$$
$$z = 14.3/59 = 0.242.$$

Introducing these values into Eqs. (6.117) and (6.118) gives $F = 1.0532$ and $E = 1.0557$.

From Table II.3, at 0.0253 eV, $\Sigma_{aM} = 0.0002728$ cm^{-1} and $\Sigma_{aF} = 0.3668$ cm^{-1}. Also, $V_M/V_F = (b^2 - a^2)/a^2 = 195.6$. Substituting into Eq. (6.114) then gives

$$\frac{1}{f} = \frac{0.0002728 \times 195.6}{0.3668} \times 1.0532 + 1.0557 = 1.2089,$$

so that

$$f = 0.8272. \text{ [\textit{Ans.}]}$$

* This case is a Square lattice; but the Homework is Hexagonal lattice.