Exercise 262

Let V and W be vector spaces over a field F. Let $\alpha \in \mathrm{Hom}(V, W)$ and $\beta \in \mathrm{Hom}(W, V)$ satisfy the condition that $\alpha\beta\alpha = \alpha$. If $w \in \mathrm{im}(\alpha)$, show tha $\alpha^{-1}(w) = \{\beta(w) + v - \beta\alpha(v) \mid v \in V\}$.

Exercise 269

Let $\alpha : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$\alpha : \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mapsto \begin{bmatrix} a+b+c \\ -a-c \\ b \end{bmatrix}.$$

Find $\ker(\alpha)$ and $\mathrm{im}(\alpha)$.

Exercise 277

Let V be a finite-dimensional vector space over a field F and let $\alpha, \beta \in \mathrm{Hom}(V, V)$ be linear transformations satisfying $\mathrm{im}(\alpha) + \mathrm{im}(\beta) = V = \ker(\alpha) + \ker(\beta)$. Show that $\mathrm{im}(\alpha) \cap \mathrm{im}(\beta) = \{0_V\} = \ker(\alpha) \cap \ker(\beta)$.

Exercise 320

Let V be a finitely-generated vector space over a field F and let $\alpha \in \mathrm{End}(V)$. Show that α is not monic if and only if there exists an endomorphism $\beta \neq \sigma_0$ of V satisfying $\alpha\beta = \sigma_0$.

Exercise 321

Let V be a vector space over a field F and let $\alpha \in \mathrm{End}(V)$. Show that $\ker(\alpha) = \ker(\alpha^2)$ if and only if $\ker(\alpha)$ and $\mathrm{im}(\alpha)$ are disjoint.

Exercise 379

Let α and β be endomorphisms of a vector space V over a field F satisfying $\alpha\beta = \beta\alpha$. Is $\ker(\alpha)$ invariant under β?

Exercise 385

Let V be a vector space over a field F and let W and Y be subspaces of V satisfying $W + Y = V$. Let Y' be a complement of Y in V and let Y'' be a complement of $W \cap Y$ in W. Show that $Y' \cong Y''$.

Exercise 387

Let V be a vector space over F and let $\alpha, \beta \in \mathrm{End}(V)$. Show that α and β are projections satisfying $\ker(\alpha) = \ker(\beta)$ if and only if $\alpha\beta = \alpha$ and $\beta\alpha = \beta$.

Exercise 421

Let $B = \{1 + i, 2 + i\}$, which is a basis for $\mathbb{C}$ as a vector space over $\mathbb{R}$. Let α be the endomorphism of this space defined by $\alpha : z \mapsto \overline{z}$. Find $\Phi_{BB}(\alpha)$.

Exercise 431

Find a nonzero matrix A in $\mathcal{M}_{2\times 2}(\mathbb{R})$ satisfying $v \odot Av = 0$ for all $v \in \mathbb{R}^2$.