

- Neatly write solutions to the following questions on separate paper.
- To receive full credit, you *MUST* write out all steps (correctly) to fully support your answers.
- All answers should be left in *EXACT* form unless otherwise stated.
- Approximately one third of your grade is based on clarity, neatness, justification, organization, and use of correct vocabulary and notation in your work (see communication rubric on the back of this page.)
- Your goal in answering these questions should be to demonstrate a clear, deep understanding of the concepts studied in this class, as well as demonstrate your connections of those ideas to related material.
- Think of your audience as if you were writing an example for a textbook for this class.
- If your work appears to be copied from *any* source, you will receive a score of zero on this entire take-home exam. Further grade or disciplinary penalties may also be assessed.

- ) 1. Look up the Fundamental Theorem of Space Curves in at least two different reliable sources. Print out or photocopy the theorem as stated in your sources (include a citation of the sources). Look up any words in the statement of the theorem you do not understand. Then in your own words explain what the theorem says. Describe any differences in the statement of the theorem from your sources.
- s) 2. Compare and contrast the *paths* and the *nature of the motion along the paths* for six different objects moving according to the following position functions, where  $t$  is a time parameter. Thoroughly justify your claims.
- $$\mathbf{r}_1(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}$$
- $$\mathbf{r}_2(t) = (2 + \cos t) \mathbf{i} + (2 + \sin t) \mathbf{j} + t \mathbf{k}$$
- $$\mathbf{r}_3(t) = \cos 5t \mathbf{i} + \sin 5t \mathbf{j} + 5t \mathbf{k}$$
- $$\mathbf{r}_4(t) = \cos(100\pi - t) \mathbf{i} + \sin(100\pi - t) \mathbf{j} + (100\pi - t) \mathbf{k}$$
- $$\mathbf{r}_5(t) = \cos t \mathbf{i} - \sin t \mathbf{j} - t \mathbf{k}$$
- $$\mathbf{r}_6(t) = \cos t^2 \mathbf{i} + \sin t^2 \mathbf{j} + t^2 \mathbf{k}$$
- ) 3. Prove that  $\mathbf{T}$  and  $\mathbf{N}$  are orthogonal. To do this, it might be helpful to use the following:
- a.  $\mathbf{w} \cdot \mathbf{w} = |\mathbf{w}|^2$  for any vector  $\mathbf{w}$
  - b.  $|\mathbf{T}| = 1$
  - c. There is a product rule for derivatives of dot products of vector-valued functions that works the way you should expect it to work. See page 730 of your book.