Solving for the loading factor we get

$$\sum_{k=1}^{M}\left(\frac{1}{1+\frac{W}{R_k}\frac{1}{\xi_k}}\right) \le 1 - \frac{I_{\text{inter}} + N_0 W}{\min\limits_{1\le i\le M}\left[P_{t,\max} G_{1,i}\left(1+\frac{W}{R_i}\frac{1}{\xi_i}\right)\right]}. \tag{5.90}$$

It can be seen that the loading in (5.90) is limited by the connection having the lowest link gain. Hence, in a DS-CDMA with best-effort services, the total cell throughput is maximized by allocating the higher data rates to the users having the best link gains. With such rate allocation, the term $\min\limits_{1\le i\le M}\left[G_{1,i}\left(1+\frac{W}{R_i}\frac{1}{\xi_i}\right)\right]$ is maximized and is directly translated into a higher cell throughput.

For the downlink case, the received signal-energy-to-interference-power-spectral-density ratio in a multi-service CDMA can be written as

$$\left(\frac{E_b}{I_0}\right)_m = \frac{W}{R_m}\frac{\phi_m P G_{1,m}}{\theta_m(1-\phi_m)PG_{1,m} + \sum_{k=2}^{B} PG_{k,m} + N_0 W} \ge \xi_i \tag{5.91}$$

with $\sum_{m=1}^{M}\phi_m \le 1$, $\xi_m \in \{\xi_{t,1}, \xi_{t,2}, \ldots, \xi_{t,K}\}$, and $R_m \in \{R_{t,1}, R_{t,2}, \ldots, R_{t,K}\}$.

Solving for the fraction of power needed for mobile user m that ensures its quality of service we get

$$\phi_m \ge \frac{1}{\theta_m + \frac{W}{R_m}\frac{1}{\xi_m}}\left(\theta_m + f_{\text{DL},m} + \frac{N_0 W}{PG_{1,m}}\right), \tag{5.92}$$

which is inversely proportional to the link gain $G_{1,m}$, and proportional to the QoS indicator $R_m\xi_m$ and the interference factor $f_{\text{DL},m}$. Hence, in a multi-service DS-CDMA with best-effort services, higher rates should be allocated to mobile users having the higher link gains (closer to the base station).

Exercises

5.1 Consider a cellular system with a total of 120 channels. The system is modeled by a regular hexagonal grid with the base stations placed in the corners of each cell. The base stations use directional antennas. Assume that the base stations use ideal sectorized antennas with 60° lobe width, i.e. every cell is divided into two halves requiring two channel groups. The path loss exponent is 4. The SIR requirement for good signal quality is 15 dB and thermal noise can be neglected. Compare the capacity (measured as channels/unit area) of this system with the two following systems:

a) A system with 120° sector antennas.
b) A system with omni-directional antennas and base stations in the center of the cell.

5.2 One way to improve the capacity of cellular systems is to employ a two-channel bandwidth scheme, where a hexagonal cell is divided into two concentric hexagons as shown in Figure 5.19. The inner hexagon is serviced by 15 kHz channels, while the outer is serviced by 30 kHz channels. Suppose that the 30 kHz channels require an

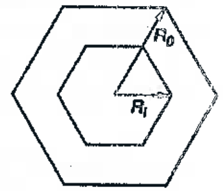

Figure 5.19 Geometry of Problem 5.2.

18 dB threshold to maintain acceptable radio link quality, while the 15 kHz channels require 24 dB. The path loss exponent is 4.

a) Consider the downlink and assume a fourth-law path loss model. Determine the ratios, $\Delta_0 = D/R_0$ and $\Delta_i = D/R_i$.
b) Determine the ratio of the inner and outer hexagonal areas, A_i/A_0.
c) Let N_i and N_0 be the number of users allocated to the inner and outer portions of the cell respectively, and assume that the channels are allocated such that $N_i/N_0 = A_i/(A_0 - A_i)$. Determine the increase in capacity (channel per cell) over a conventional one-channel bandwidth system that uses only 30 kHz channels, if the system uses a 7-cell reuse cluster.

Consider only the first tier of interference and neglect the thermal noise.

5.3 A cellular telephony system uses a static channel allocation with $\eta = 25$ channels per cell. The arrival process of new calls is modeled as a Poisson process with arrival rate $\lambda = 180$ calls/h/km^2. Every call has an exponentially distributed duration with an average $1/\mu = 2.5$ minutes.

a) Determine the traffic intensity (erlang/cell) if the cell radius is $R = 1$ km.
b) What is the blocking probability at this traffic load?
c) Determine the channel assignment failure rate at the traffic load if we can assume that (almost) all calls are handled by the system.

5.4 A wireless cellular system has a total of 100 channels with all base stations placed on a highway. The base stations are equidistant with a distance of 2 km between two neighboring base stations. The base stations employ directional antennas and radiate power in one direction only. Assume that the propagation loss increases with the fourth power of the distance and the thermal noise can be neglected.

a) Determine the capacity (in channels/km) that can be achieved if the SIR requirement is 20 dB.
b) Determine the blocking probability as a function of the relative traffic load (erlang/cell/channel).
c) Determine the channel assignment failure rate as a function of the relative load (active terminals/cell/channel).

5.5 Consider a hexagonal cellular system where base stations have been mounted in the middle of the cells. Each base station uses an omni-directional antenna and an

5.16 Consider the uplink of a single-cell CDMA system with a base station having an omni-directional antenna placed in the middle of the cell. The mobile users are uniformly distributed over the cell area. The required SINR for good signal quality is $\gamma_t = -19.96$ dB. The propagation path loss is modeled as

$$L_P(r) = 76 + 40\log_{10}(r) \quad \text{dB},$$

where r, in km, is the distance to the base station.

The terminals are using an ideal constant received power control, the maximum transmission power is $p_{max} = 23$ dBm, and the noise power is $N_0W = -33$ dBm.

a) Determine the outage probability for a user placed at a distance r from the base station as a function of the number of active users M and r.
b) Derive a relation showing the tradeoff between capacity (number of users) and coverage r, and make a plot of the obtained results.

5.17 In order to provide radio communication at a horse-race event, a base station is placed in the middle of the arena, so that any spectator is at exactly $d = 500$ m from the base station. However, each spectator experiences a log-normal distributed fading with standard deviation $\sigma = 5.102$ dB; assume that the fading values experienced by any two spectators are uncorrelated. On the uplink, the system uses CDMA with ideal SIR-target power control and provides two types of services. The voice service requires an SIR target $\gamma_v = -20$ dB, while the data service requires $\gamma_d = -14$ dB. One out of five users uses data services. If only one terminal were active, transmitting with the maximum allowed power, the median would be $\gamma_0 = 6$ dB.

a) Find an expression for the largest $\frac{1}{\eta}$ so that the users have access to the data service with probability 97.5%.
b) How many users can be served in this single-cell system under the conditions at point a)?

5.18 Consider the uplink in a single hexagonal cell CDMA system, with a cell radius $R = 1075$ m. The users are uniformly distributed over the service area with 7 users/km^2. The required signal-energy-to-noise-power-spectral-density ratio for good signal quality is $(E_b/N_0) = 7$ dB and the system processing gain is 128.

The path gain is modeled as $G(d) = c_0/d^\alpha$, with d in meters, $c_0 = -30$ dB, $\alpha = 3.5$, and the noise power is $N = -103$ dBm.

a) If ideal constant-received-power control is used, determine the minimum required received power that ensures the desired (E_b/N_0) target for all the users.
b) If the maximum transmitted power is limited to $P_{max} = 100$ mW, determine the number of users that cannot reach the required signal quality.
c) How many users can the system support such that good signal quality is ensured for all the users anywhere within the cell?

5.19 Consider the uplink in a single-cell CDMA system, with a cell radius $\rho = 1000$ m. There are two service types in the system, corresponding to two different end-user bit rates. Both services require the same demodulated signal quality $E_b/N_0 = 5$ dB, but use different processing gains $P_{g,1} = 21$ dB and $P_{g,2} = 25$ dB (roughly corresponding to a 38 kbps data service and a 12.2 kbps voice service respectively). Each terminal uses ideal (perfect) power control to reach the desired SIR target; the maximum transmission power is $P_{max} = 100$ mW.

a) What is the highest possible system load allowed in the system in order to provide a high bit rate service to a user located at the cell border? (Express it as load L, η or P_{tot}/N). The path gain is modeled as $G(d) = c_0/d^\alpha$, with $c_0 = -24$ dB and $\alpha = 3.5$, and the noise power is $N = -103$ dBm.
b) Assume that the cell contains only one user with high bit rate service and a number of M users with low bit rate service. What is the largest number M of low bit rate users for which the high bit rate user can be served at the cell border?
c) Repeat the computations and find M when the processing gain for the high bit rate service is $P_{g,1} = 14$ dB.

5.20 A WCDMA system with a chip rate of 3.84 Mcps and an information data rate of 9.6 kbps serves two kinds of users: *gold users* who require a bit-energy-to-interference-spectral-density ratio threshold of 10 dB and *silver users* who require a bit-energy-to-interference-spectral-density ratio threshold of only 7 dB.

a) Consider the uplink situation in a single-cell system and denote by x the number of gold users and by y the number of silver users. Derive the required received power, at the base station, from each terminal as a function of x and y.
b) Show that the capacity of this WCDMA system, denoted by $\eta = x + y$, is a surface in the (x,y) signal space. Make a plot of this signal space and identify the capacity region for which the signal quality can be maintained for all active users.

5.21 Consider the uplink of a DS-CDMA multi-cellular system where we have a total of N users per cell, a total bandwidth of W hertz, and a user data rate of R bits/s. The base station receiver employs nonlinear successive cancelation where users are detected successively and canceled from the received signal one by one. The strongest user is detected first, remodulated, and then canceled from the received signal. The obtained signal is then used to detect the next strongest user and so on.

The required bit-energy-to-interference-spectral-density ratio for good signal quality is denoted ξ_t and is the same for all users.

a) Assuming that there is no error propagation along the stages, compute the required powers for the different users that achieve the required threshold for all the users as a function of ξ_t, W, R, N_0 and I_e (inter-cell interference).
b) Derive an upper bound on the system capacity, N, if the total received power from the users within the cell is not to exceed 3 dB above the noise level $N_0W + I_e$.

5.22 A WCDMA system with a chip rate of 3.84 Mcps supports two types of users. A low rate type with a data rate of 9.6 kbps and a high rate type with a data rate of 480 kbps. To provide these rates, the system uses orthogonal variable length (OVL) spreading codes as channelization codes and long random codes as scrambling codes.