



In the previous example, we selected and arranged all the objects, that is $n = r$. In many cases, only some objects are selected and arranged from the n possible objects. We explain the details of this application in the following example.

Example

Betts Machine Shop Inc. has eight screw machines but only three spaces available in the production area for the machines. In how many different ways can the eight machines be arranged in the three spaces available?

Solution

There are eight possibilities for the first available space in the production area, seven for the second space (one has been used up), and six for the third space. Thus:

$$(8)(7)(6) = 336,$$

that is, there are a total of 336 different possible arrangements. This could also be found by using formula (5-9). If $n = 8$ machines, and $r = 3$ spaces available, the formula leads to

$${}_nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{(8)(7)(6)5!}{5!} = 336$$

The Combination Formula

If the order of the selected objects is *not* important, any selection is called a **combination**. The formula to count the number of r object combinations from a set of n objects is:

COMBINATION FORMULA

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

[5-10]

For example, if executives Able, Baker, and Chauncy are to be chosen as a committee to negotiate a merger, there is only one possible combination of these three; the committee of Able, Baker, and Chauncy is the same as the committee of Baker, Chauncy, and Able. Using the combination formula:

$${}_nC_r = \frac{n!}{r!(n-r)!} = \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1(1)} = 1$$

Example

The marketing department has been given the assignment of designing color codes for the 42 different lines of compact disks sold by Goody Records. Three colors are to be used on each CD, but a combination of three colors used for one CD cannot be rearranged and used to identify a different CD. This means that if green, yellow, and violet were used to identify one line, then yellow, green, and violet (or any other combination of these three colors) cannot be used to identify another line. Would seven colors taken three at a time be adequate to color-code the 42 lines?

Solution

According to formula (5-10), there are 35 combinations, found by

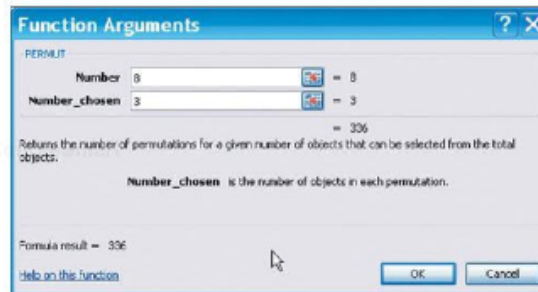
$${}_nC_3 = \frac{n!}{r!(n-r)!} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = 35$$

The seven colors taken three at a time (i.e., three colors to a line) would not be adequate to color-code the 42 different lines because they would provide only 35 combinations. Eight colors taken three at a time would give 56 different combinations. This would be more than adequate to color-code the 42 different lines.



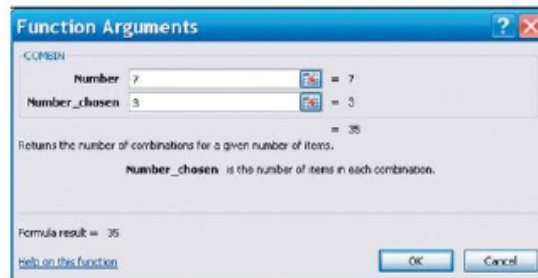


When the number of permutations or combinations is large, the calculations are tedious. Computer software and handheld calculators have "functions" to compute these numbers. The Excel output for the location of the eight screw machines in the production area of Betts Machine Shop Inc. is shown below. There are a total of 336 arrangements.



The dialog box is titled "Function Arguments" and shows the "PERMUT" function. The "Number" field is set to 8 and the "Number_chosen" field is set to 3. The formula result is 336. The description states: "Returns the number of permutations for a given number of objects that can be selected from the total objects. Number_chosen is the number of objects in each permutation." Buttons for "OK" and "Cancel" are at the bottom right.

Below is the output for the color codes at Goody Records. Three colors are chosen from among seven possible. The number of combinations possible is 35.



The dialog box is titled "Function Arguments" and shows the "COMBIN" function. The "Number" field is set to 7 and the "Number_chosen" field is set to 3. The formula result is 35. The description states: "Returns the number of combinations for a given number of items. Number_chosen is the number of items in each combination." Buttons for "OK" and "Cancel" are at the bottom right.

Self-Review 5-11



1. A musician wants to write a score based on only five chords: B-flat, C, D, E, and G. However, only three chords out of the five will be used in succession, such as C, B-flat, and E. Repetitions, such as B-flat, B-flat, and E, will not be permitted.
 - (a) How many permutations of the five chords, taken three at a time, are possible?
 - (b) Using formula (5-9), how many permutations are possible?
2. The 10 numbers 0 through 9 are to be used in code groups of four to identify an item of clothing. Code 1063 might identify a blue blouse, size medium; the code group 2031 might identify a pair of pants, size 18; and so on. Repetitions of numbers are not permitted. That is, the same number cannot be used twice (or more) in a total sequence. For example, 2256, 2562, or 5559 would not be permitted. How many different code groups can be designed?
3. In the above example involving Goody Records, we said that eight colors taken three at a time would give 56 different combinations.
 - (a) Use formula (5-10) to show this is true.
 - (b) As an alternative plan for color-coding the 42 different lines, it has been suggested that only two colors be placed on a disk. Would 10 colors be adequate to color-code





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the 42 different lines? (Again, a combination of two colors could be used only once—that is, if pink and blue were coded for one line, blue and pink could not be used to identify a different line.)

4. In a lottery game, three numbers are randomly selected from a tumbler of balls numbered 1 through 50.
- How many permutations are possible?
 - How many combinations are possible?

Exercises

connect

39. Solve the following:
- $40!/35!$
 - ${}_7P_4$
 - ${}_8C_9$
40. Solve the following:
- $20!/17!$
 - ${}_9P_3$
 - ${}_7C_2$
41. A pollster randomly selected 4 of 10 available people. How many different groups of 4 are possible?
42. A telephone number consists of seven digits, the first three representing the exchange. How many different telephone numbers are possible within the 537 exchange?
43. An overnight express company must include five cities on its route. How many different routes are possible, assuming that it does not matter in which order the cities are included in the routing?
44. A representative of the Environmental Protection Agency (EPA) wants to select samples from 10 landfills. The director has 15 landfills from which she can collect samples. How many different samples are possible?
45. A national pollster has developed 15 questions designed to rate the performance of the president of the United States. The pollster will select 10 of these questions. How many different arrangements are there for the order of the 10 selected questions?
46. A company is creating three new divisions and seven managers are eligible to be appointed head of a division. How many different ways could the three new heads be appointed? Hint: Assume the division assignment makes a difference.

Chapter Summary

- A probability is a value between 0 and 1 inclusive that represents the likelihood a particular event will happen.
 - An experiment is the observation of some activity or the act of taking some measurement.
 - An outcome is a particular result of an experiment.
 - An event is the collection of one or more outcomes of an experiment.
- There are three definitions of probability.
 - The classical definition applies when there are n equally likely outcomes to an experiment.
 - The empirical definition occurs when the number of times an event happens is divided by the number of observations.
 - A subjective probability is based on whatever information is available.
- Two events are mutually exclusive if by virtue of one event happening the other cannot happen.
- Events are independent if the occurrence of one event does not affect the occurrence of another event.
- The rules of addition refer to the union of events.

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**Statistics in Action**

Government statistics show there are about 1.7 automobile-caused fatalities for every 100,000,000 vehicle-miles. If you drive 1 mile to the store to buy your lottery ticket and then return home, you have driven 2 miles. Thus the probability that you will join this statistical group on your next 2 mile round trip is $2 \times 1.7 / 100,000,000 = 0.000000034$. This can also be stated as "One in 29,411,765." Thus if you drive to the store to buy your Powerball ticket, your chance of being killed (or killing someone else) is more than 4 times greater than the chance that you will win the Powerball Jackpot, one chance in 120,526,770. <http://www.durangoball.com/Powerball Odds.html>

A Survey of Probability Concepts

177

- A. The special rule of addition is used when events are mutually exclusive.

$$P(A \text{ or } B) = P(A) + P(B) \quad [5-2]$$

- B. The general rule of addition is used when the events are not mutually exclusive.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad [5-4]$$

- C. The complement rule is used to determine the probability of an event happening by subtracting the probability of the event not happening from 1.

$$P(A) = 1 - P(\sim A) \quad [5-3]$$

- VI. The rules of multiplication refer to the product of events.

- A. The special rule of multiplication refers to events that are independent.

$$P(A \text{ and } B) = P(A)P(B) \quad [5-5]$$

- B. The general rule of multiplication refers to events that are not independent.

$$P(A \text{ and } B) = P(A)P(B|A) \quad [5-6]$$

- C. A joint probability is the likelihood that two or more events will happen at the same time.

- D. A conditional probability is the likelihood that an event will happen, given that another event has already happened.

- E. Bayes' theorem is a method of revising a probability, given that additional information is obtained. For two mutually exclusive and collectively exhaustive events:

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \quad [5-7]$$

- VII. There are three counting rules that are useful in determining the number of outcomes in an experiment.

- A. The multiplication rule states that if there are m ways one event can happen and n ways another event can happen, then there are mn ways the two events can happen.

$$\text{Number of arrangements} = (m)(n) \quad [5-8]$$

- B. A permutation is an arrangement in which the order of the objects selected from a specific pool of objects is important.

$${}_nP_r = \frac{n!}{(n-r)!} \quad [5-9]$$

- C. A combination is an arrangement where the order of the objects selected from a specific pool of objects is not important.

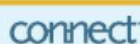
$${}_nC_r = \frac{n!}{r!(n-r)!} \quad [5-10]$$

Pronunciation Key


SYMBOL	MEANING	PRONUNCIATION
$P(A)$	Probability of A	P of A
$P(\sim A)$	Probability of not A	P of not A
$P(A \text{ and } B)$	Probability of A and B	P of A and B
$P(A \text{ or } B)$	Probability of A or B	P of A or B
$P(A B)$	Probability of A given B has happened	P of A given B
${}_nP_r$	Permutation of n items selected r at a time	Pnr
${}_nC_r$	Combination of n items selected r at a time	Cnr

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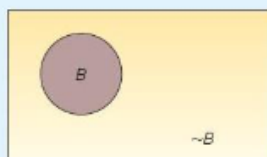


Chapter Exercises

47. The marketing research department at Pepsico plans to survey teenagers about a newly developed soft drink. Each will be asked to compare it with his or her favorite soft drink.
- What is the experiment?
 - What is one possible event?
48. The number of times a particular event occurred in the past is divided by the number of occurrences. What is this approach to probability called?
49. The probability that the cause and the cure for all cancers will be discovered before the year 2020 is .20. What viewpoint of probability does this statement illustrate?
50. Berdine's Chicken Factory has several stores in the Hilton Head, South Carolina, area. When interviewing applicants for server positions, the owner would like to include information on the amount of tip a server can expect to earn per check (or bill). A study of 500 recent checks indicated the server earned the following amounts in tips per 8-hour shift. 

Amount of Tip	Number
\$0 up to \$ 20	200
20 up to 50	100
50 up to 100	75
100 up to 200	75
200 or more	50
Total	500

- What is the probability of a tip of \$200 or more?
 - Are the categories "\$0 up to \$20," "\$20 up to \$50," and so on considered mutually exclusive?
 - If the probabilities associated with each outcome were totaled, what would that total be?
 - What is the probability of a tip of up to \$50?
 - What is the probability of a tip of less than \$200?
51. Winning all three "Triple Crown" races is considered the greatest feat of a pedigree racehorse. After a successful Kentucky Derby, Big Brown is a 1 to 2 favorite to win the Preakness Stakes.
- If he is a 1 to 2 favorite to win the Belmont Stakes as well, what is his probability of winning the Triple Crown?
 - What do his chances for the Preakness Stakes have to be in order for him to be "even money" to earn the Triple Crown?
52. The first card selected from a standard 52-card deck is a king.
- If it is returned to the deck, what is the probability that a king will be drawn on the second selection?
 - If the king is not replaced, what is the probability that a king will be drawn on the second selection?
 - What is the probability that a king will be selected on the first draw from the deck and another king on the second draw (assuming that the first king was not replaced)?
53. Armco, a manufacturer of traffic light systems, found that under accelerated-life tests, 95 percent of the newly developed systems lasted 3 years before failing to change signals properly.
- If a city purchased four of these systems, what is the probability all four systems would operate properly for at least 3 years?
 - Which rule of probability does this illustrate?
 - Using letters to represent the four systems, write an equation to show how you arrived at the answer to part (a).
54. Refer to the following picture.





A Survey of Probability Concepts

179


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- a. What is the picture called?
b. What rule of probability is illustrated?
c. B represents the event of choosing a family that receives welfare payments. What does $P(B) + P(\sim B)$ equal?
55. In a management trainee program at Claremont Enterprises, 80 percent of the trainees are female and 20 percent male. Ninety percent of the females attended college, and 78 percent of the males attended college.
a. A management trainee is selected at random. What is the probability that the person selected is a female who did not attend college?
b. Are gender and attending college independent? Why?
c. Construct a tree diagram showing all the probabilities, conditional probabilities, and joint probabilities.
d. Do the joint probabilities total 1.00? Why?
56. Assume the likelihood that any flight on Delta Airlines arrives within 15 minutes of the scheduled time is .90. We select four flights from yesterday for study.
a. What is the likelihood all four of the selected flights arrived within 15 minutes of the scheduled time?
b. What is the likelihood that none of the selected flights arrived within 15 minutes of the scheduled time?
c. What is the likelihood at least one of the selected flights did not arrive within 15 minutes of the scheduled time?
57. There are 100 employees at Kiddie Carts International. Fifty-seven of the employees are production workers, 40 are supervisors, 2 are secretaries, and the remaining employee is the president. Suppose an employee is selected:
a. What is the probability the selected employee is a production worker?
b. What is the probability the selected employee is either a production worker or a supervisor?
c. Refer to part (b). Are these events mutually exclusive?
d. What is the probability the selected employee is neither a production worker nor a supervisor?
58. Joe Mauer of the Minnesota Twins had the highest batting average in the 2009 Major League Baseball season. His average was .365. So assume the probability of getting a hit is .365 for each time he batted. In a particular game, assume he batted three times.
a. This is an example of what type of probability?
b. What is the probability of getting three hits in a particular game?
c. What is the probability of not getting any hits in a game?
d. What is the probability of getting at least one hit?
59. Four sports teams remain in a single-elimination playoff competition. If one team is favored in its semi-final match by odds of 2 to 1 and another squad is favored in its contest by odds of 3 to 1. What is the probability that:
a. Both favored teams win their games?
b. Neither favored team wins its game?
c. At least one of the favored teams wins its game?
60. There are three clues labeled "daily double" on the game show Jeopardy. If three equally matched contenders play, what is the probability that:
a. A single contestant finds all three "daily doubles"?
b. The returning champion gets all three of the "daily doubles"?
c. Each of the players selects precisely one of the "daily doubles"?
61. Brooks Insurance Inc. wishes to offer life insurance to men age 60 via the Internet. Mortality tables indicate the likelihood of a 60-year-old man surviving another year is .98. If the policy is offered to five men age 60:
a. What is the probability all five men survive the year?
b. What is the probability at least one does not survive?
62. Forty percent of the homes constructed in the Quail Creek area include a security system. Three homes are selected at random:
a. What is the probability all three of the selected homes have a security system?
b. What is the probability none of the three selected homes have a security system?
c. What is the probability at least one of the selected homes has a security system?
d. Did you assume the events to be dependent or independent?
63. Refer to Exercise 62, but assume there are 10 homes in the Quail Creek area and 4 of them have a security system. Three homes are selected at random:
a. What is the probability all three of the selected homes have a security system?
b. What is the probability none of the three selected homes have a security system?
c. What is the probability at least one of the selected homes has a security system?
d. Did you assume the events to be dependent or independent?

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64. There are 20 families living in the Willbrook Farms Development. Of these families, 10 prepared their own federal income taxes for last year, 7 had their taxes prepared by a local professional, and the remaining 3 by H&R Block.
- What is the probability of selecting a family that prepared their own taxes?
 - What is the probability of selecting two families, both of which prepared their own taxes?
 - What is the probability of selecting three families, all of which prepared their own taxes?
 - What is the probability of selecting two families, neither of which had their taxes prepared by H&R Block?
65. The board of directors of Saner Automatic Door Company consists of 12 members, 3 of whom are women. A new policy and procedures manual is to be written for the company. A committee of 3 is randomly selected from the board to do the writing.
- What is the probability that all members of the committee are men?
 - What is the probability that at least 1 member of the committee is a woman?
66. A recent survey reported in *BusinessWeek* dealt with the salaries of CEOs at large corporations and whether company shareholders made money or lost money. 

	CEO Paid More Than \$1 Million	CEO Paid Less Than \$1 Million	Total
Shareholders made money	2	11	13
Shareholders lost money	4	3	7
Total	6	14	20

If a company is randomly selected from the list of 20 studied, what is the probability:

- The CEO made more than \$1 million?
 - The CEO made more than \$1 million or the shareholders lost money?
 - The CEO made more than \$1 million given the shareholders lost money?
 - Of selecting 2 CEOs and finding they both made more than \$1 million?
67. Althoff and Roll, an investment firm in Augusta, Georgia, advertises extensively in the *Augusta Morning Gazette*, the newspaper serving the region. The *Gazette* marketing staff estimates that 60 percent of Althoff and Roll's potential market read the newspaper. It is further estimated that 65 percent of those who read the *Gazette* remember the Althoff and Roll advertisement.
- What percent of the investment firm's potential market sees and remembers the advertisement?
 - What percent of the investment firm's potential market sees, but does not remember the advertisement?
68. An Internet company located in Southern California has season tickets to the Los Angeles Lakers basketball games. The company president always invites one of the four vice presidents to attend games with him, and claims he selects the person to attend at random. One of the four vice presidents has not been invited to attend any of the last five Lakers home games. What is the likelihood this could be due to chance?
69. A computer-supply retailer purchased a batch of 1,000 CD-R disks and attempted to format them for a particular application. There were 857 perfect CDs, 112 CDs were usable but had bad sectors, and the remainder could not be used at all.
- What is the probability a randomly chosen CD is not perfect?
 - If the disk is not perfect, what is the probability it cannot be used at all?
70. An investor purchased 100 shares of Fifth Third Bank stock and 100 shares of Santee Electric Cooperative stock. The probability the bank stock will appreciate over a year is .70. The probability the electric utility will increase over the same period is .60.
- What is the probability both stocks appreciate during the period?
 - What is the probability the bank stock appreciates but the utility does not?
 - What is the probability at least one of the stocks appreciates?
71. Flashner Marketing Research Inc. specializes in providing assessments of the prospects for women's apparel shops in shopping malls. Al Flashner, president, reports that he assesses the prospects as good, fair, or poor. Records from previous assessments show that 60 percent of the time the prospects were rated as good, 30 percent of the time fair, and 10 percent of the time poor. Of those rated good, 60 percent made a profit the first year; of those rated fair, 60 percent made a profit the first year; and of those rated poor, 20 percent made a profit the first year. Connie's Apparel was one of Flashner's clients. Connie's Apparel made a profit last year. What is the probability that it was given an original rating of poor?
72. Two boxes of men's Old Navy shirts were received from the factory. Box 1 contained 25 mesh polo shirts and 15 Super-T shirts. Box 2 contained 30 mesh polo shirts and 10





A Survey of Probability Concepts

181

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- Super-T shirts. One of the boxes was selected at random, and a shirt was chosen at random from that box to be inspected. The shirt was a mesh polo shirt. Given this information, what is the probability that the mesh polo shirt came from box 1?
73. With each purchase of a large pizza at Tony's Pizza, the customer receives a coupon that can be scratched to see if a prize will be awarded. The odds of winning a free soft drink are 1 in 10, and the odds of winning a free large pizza are 1 in 50. You plan to eat lunch tomorrow at Tony's. What is the probability:
- That you will win either a large pizza or a soft drink?
 - That you will not win a prize?
 - That you will not win a prize on three consecutive visits to Tony's?
 - That you will win at least one prize on one of your next three visits to Tony's?
74. For the daily lottery game in Illinois, participants select three numbers between 0 and 9. A number cannot be selected more than once, so a winning ticket could be, say, 307 but not 337. Purchasing one ticket allows you to select one set of numbers. The winning numbers are announced on TV each night.
- How many different outcomes (three-digit numbers) are possible?
 - If you purchase a ticket for the game tonight, what is the likelihood you will win?
 - Suppose you purchase three tickets for tonight's drawing and select a different number for each ticket. What is the probability that you will not win with any of the tickets?
75. Several years ago, Wendy's Hamburgers advertised that there are 256 different ways to order your hamburger. You may choose to have, or omit, any combination of the following on your hamburger: mustard, ketchup, onion, pickle, tomato, relish, mayonnaise, and lettuce. Is the advertisement correct? Show how you arrive at your answer.
76. It was found that 60 percent of the tourists to China visited the Forbidden City, the Temple of Heaven, the Great Wall, and other historical sites in or near Beijing. Forty percent visited Xi'an with its magnificent terracotta soldiers, horses, and chariots, which lay buried for over 2,000 years. Thirty percent of the tourists went to both Beijing and Xi'an. What is the probability that a tourist visited at least one of these places?
77. A new chewing gum has been developed that is helpful to those who want to stop smoking. If 60 percent of those people chewing the gum are successful in stopping smoking, what is the probability that in a group of four smokers using the gum at least one quits smoking?
78. Reynolds Construction Company has agreed not to erect all "look-alike" homes in a new subdivision. Five exterior designs are offered to potential home buyers. The builder has standardized three interior plans that can be incorporated in any of the five exteriors. How many different ways can the exterior and interior plans be offered to potential home buyers?
79. A new sports car model has defective brakes 15 percent of the time and a defective steering mechanism 5 percent of the time. Let's assume (and hope) that these problems occur independently. If one or the other of these problems is present, the car is called a "lemon." If both of these problems are present, the car is a "hazard." Your instructor purchased one of these cars yesterday. What is the probability it is:
- A lemon?
 - A hazard?
80. The state of Maryland has license plates with three numbers followed by three letters. How many different license plates are possible?
81. There are four people being considered for the position of chief executive officer of Dalton Enterprises. Three of the applicants are over 60 years of age. Two are female, of which only one is over 60.
- What is the probability that a candidate is over 60 and female?
 - Given that the candidate is male, what is the probability he is less than 60?
 - Given that the person is over 60, what is the probability the person is female?
82. Tim Bleckie is the owner of Bleckie Investment and Real Estate Company. The company recently purchased four tracts of land in Holly Farms Estates and six tracts in Newburg Woods. The tracts are all equally desirable and sell for about the same amount.
- What is the probability that the next two tracts sold will be in Newburg Woods?
 - What is the probability that of the next four sold at least one will be in Holly Farms?
 - Are these events independent or dependent?
83. A computer password consists of four characters. The characters can be one of the 26 letters of the alphabet. Each character may be used more than once. How many different passwords are possible?
84. A case of 24 cans contains 1 can that is contaminated. Three cans are to be chosen randomly for testing.
- How many different combinations of 3 cans could be selected?
 - What is the probability that the contaminated can is selected for testing?

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85. A puzzle in the newspaper presents a matching problem. The names of 10 U.S. presidents are listed in one column, and their vice presidents are listed in random order in the second column. The puzzle asks the reader to match each president with his vice president. If you make the matches randomly, how many matches are possible? What is the probability all 10 of your matches are correct?
86. Two components, A and B , operate in series. Being in series means that for the system to operate, both components A and B must work. Assume the two components are independent. What is the probability the system works under these conditions? The probability A works is .90 and the probability B functions is also .90.
87. Horwege Electronics Inc. purchases TV picture tubes from four different suppliers. Tyson Wholesale supplies 20 percent of the tubes, Fuji Importers 30 percent, Kirkpatricks 25 percent, and Parts Inc. 25 percent. Tyson Wholesale tends to have the best quality, as only 3 percent of its tubes arrive defective. Fuji Importers' tubes are 4 percent defective, Kirkpatricks' 7 percent, and Parts Inc. are 6.5 percent defective.
- What is the overall percent defective?
 - A defective picture tube was discovered in the latest shipment. What is the probability that it came from Tyson Wholesale?
88. ABC Auto Insurance classifies drivers as good, medium, or poor risks. Drivers who apply to them for insurance fall into these three groups in the proportions 30 percent, 50 percent, and 20 percent, respectively. The probability a "good" driver will have an accident is .01, the probability a "medium" risk driver will have an accident is .03, and the probability a "poor" driver will have an accident is .10. The company sells Mr. Brophy an insurance policy and he has an accident. What is the probability Mr. Brophy is:
- A "good" driver?
 - A "medium" risk driver?
 - A "poor" driver?
89. You take a trip by air that involves three independent flights. If there is an 80 percent chance each specific leg of the trip is done on time, what is the probability all three flights arrive on time?
90. The probability a HP network server is down is .05. If you have three independent servers, what is the probability that at least one of them is operational?
91. Twenty-two percent of all liquid crystal displays (LCDs) are manufactured by Samsung. What is the probability that in a collection of three independent LCD purchases, at least one is a Samsung?

Data Set Exercises

92. Refer to the Real Estate data, which reports information on homes sold in the Goodyear, Arizona, area during the last year.
- Sort the data into a table that shows the number of homes that have a pool versus the number that don't have a pool in each of the five townships. If a home is selected at random, compute the following probabilities:
 - The home is in Township 1 or has a pool.
 - Given that it is in Township 3, that it has a pool.
 - The home has a pool and is in Township 3.
 - Sort the data into a table that shows the number of homes that have a garage attached versus those that don't in each of the five townships. If a home is selected at random, compute the following probabilities:
 - The home has a garage attached.
 - The home does not have a garage attached, given that it is in Township 5.
 - The home has a garage attached and is in Township 3.
 - The home does not have a garage attached or is in Township 2.
93. Refer to the Baseball 2009 data, which reports information on the 30 Major League Baseball teams for the 2009 season. Set up three variables:
- Divide the teams into two groups, those that had a winning season and those that did not. That is, create a variable to count the teams that won 81 games or more, and those that won 80 or less.
 - Create a new variable for attendance, using three categories: attendance less than 2.0 million, attendance of 2.0 million up to 3.0 million, and attendance of 3.0 million or more.
 - Create a variable that shows the teams that play in a stadium less than 15 years old versus one that is 15 years old or more.





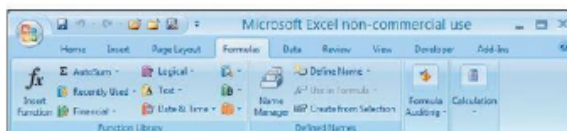
Answer the following questions.

- a. Create a table that shows the number of teams with a winning season versus those with a losing season by the three categories of attendance. If a team is selected at random, compute the following probabilities:
 1. The team had a winning season.
 2. The team had a winning season or attendance of more than 3.0 million.
 3. The team had a winning season given attendance was more than 3.0 million.
 4. The team has a winning season and attracted fewer than 2.0 million fans.
 - b. Create a table that shows the number of teams with a winning season versus those that play in new or old stadiums. If a team is selected at random, compute the following probabilities:
 1. Selecting a team with a winning season.
 2. The likelihood of selecting a team with a winning record and playing in a new stadium.
 3. The team had a winning record or played in a new stadium.
94. Refer to the data on the school buses in the Buena School District. Set up a variable that divides the age of the buses into three groups: new (less than 5 year old), medium (5 but less than 10 years), and old (10 or more years). The median maintenance cost is \$456. Based on this value, create a variable for those less than the median (low maintenance) and those more than the median (high maintenance). Finally, develop a table to show the relationship between maintenance cost and age of the bus.
- a. What percentage of the buses are new?
 - b. What percentage of the new buses have low maintenance?
 - c. What percentage of the old buses have high maintenance?
 - d. Does maintenance cost seem to be related to the age of the bus? Hint: Compare the maintenance cost of the old buses with the cost of the new buses? Would you conclude maintenance cost is independent of the age?

Software Commands

1. The Excel Commands to determine the number of permutations shown on page 175 are:

- a. Click on the **Formulas** tab in the top menu, then, on the far left, select **Insert Function** fx .



- b. In the **Insert Function** box, select **Statistical** as the category, then scroll down to **PERMUT** in the **Select a function list**. Click **OK**.
- c. In the **PERM** box after **Number**, enter 8 and in the **Number_chosen** box enter 3. The correct answer of 336 appears twice in the box.





2. The Excel Commands to determine the number of combinations shown on page 175 are:

- a. Click on the **Formulas** tab in the top menu, then, on the far left, select **Insert Function fx**.



- b. In the **Insert Function** box, select **Math & Trig** as the category, then scroll down to **COMBIN** in the **Select a function list**. Click **OK**.
c. In the **COMBIN** box after **Number**, enter 7 and in the **Number_chosen** box enter 3. The correct answer of 35 appears twice in the box.

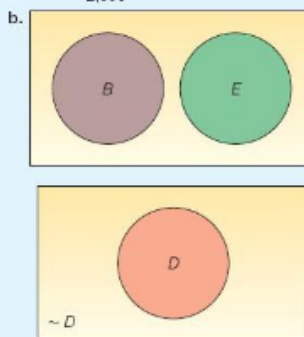


Chapter 5 Answers to Self-Review



- 5-1 a. Count the number who think the new game is playable.
b. Seventy-three players found the game playable. Many other answers are possible.
c. No. Probability cannot be greater than 1. The probability that the game, if put on the market, will be successful is 65/80, or .8125.
d. Cannot be less than 0. Perhaps a mistake in arithmetic.
e. More than half of the players testing the game liked it. (Of course, other answers are possible.)
- 5-2 1. $\frac{4 \text{ queens in deck}}{52 \text{ cards total}} = \frac{4}{52} = .0769$ Classical.
2. $\frac{182}{538} = .338$ Empirical.
3. The author's view when writing the text of the chance that the DJIA will climb to 12,000 is .25. You may be more optimistic or less optimistic. Subjective.

- 5-3 a. i. $\frac{(50 + 66)}{2,000} = .059$
ii. $1 - \frac{302}{2,000} = .849$





A Survey of Probability Concepts

185

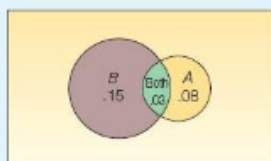
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- c. They are not complementary, but are mutually exclusive.

- 5-4 a. Need for corrective shoes is event A. Need for major dental work is event B.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= .08 + .15 - .03 \\ &= .20 \end{aligned}$$

- b. One possibility is:



5-5 $(.95)(.95)(.95)(.95) = .8145$

- 5-6 a. .002, found by:

$$\left(\frac{4}{12}\right)\left(\frac{3}{11}\right)\left(\frac{2}{10}\right)\left(\frac{1}{9}\right) = \frac{24}{11,880} = .002$$

- b. .14, found by:

$$\left(\frac{6}{12}\right)\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)\left(\frac{5}{9}\right) = \frac{1,680}{11,880} = .1414$$

- c. No, because there are other possibilities, such as three women and one man.

5-7 a. $P(B_2) = \frac{105}{200} = .525$

b. $P(A_2|B_2) = \frac{30}{105} = .286$

c. $P(A_2 \text{ or } B_2) = \frac{80}{200} + \frac{105}{200} - \frac{30}{200} = \frac{155}{200} = .775$

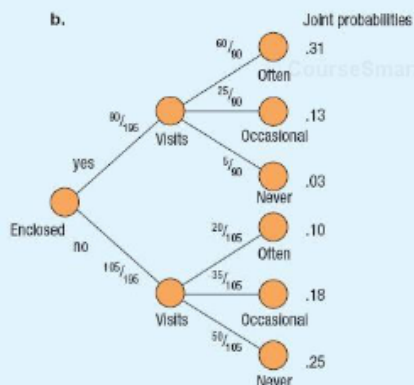
- 5-8 a. Independence requires that $P(A|B) = P(A)$. One possibility is:

$$\frac{P(\text{visit often}|\text{yes enclosed mail})}{P(\text{visit often})} =$$

Does $60/90 = 80/195$? No, the two variables are not independent.

Therefore, any joint probability in the table must be computed by using the general rule of multiplication.

- b.



5-9 a. $P(A_2|B_2) = \frac{P(A_2)P(B_2|A_2)}{P(A_1)P(B_2|A_1) + P(A_2)P(B_2|A_2) + P(A_3)P(B_2|A_3)}$

b.
$$\begin{aligned} &= \frac{(.50)(.96)}{(.30)(.97) + (.20)(.95) + (.50)(.96)} \\ &= \frac{.480}{.961} = .499 \end{aligned}$$

5-10 1. $(5)(4) = 20$

2. $(3)(2)(4)(3) = 72$

5-11 1. a. 60, found by $(5)(4)(3)$.

- b. 60, found by:

$$\frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$$

2. 5,040, found by:

$$\frac{10!}{(10-4)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

3. a. 56 is correct, found by:

$${}_6C_3 = \frac{n!}{r!(n-r)!} = \frac{6!}{3!(6-3)!} = 56$$

- b. Yes. There are 45 combinations, found by:

$${}_{10}C_2 = \frac{n!}{r!(n-r)!} = \frac{10!}{2!(10-2)!} = 45$$

4. a. ${}_{10}P_3 = \frac{50!}{(50-3)!} = 117,600$

b. ${}_{50}C_3 = \frac{50!}{3!(50-3)!} = 19,600$

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Learning Objectives

When you have completed this chapter, you will be able to:

L01 Identify the characteristics of a probability distribution.

L02 Distinguish between a discrete and a continuous random variable.

L03 Compute the mean of a probability distribution.

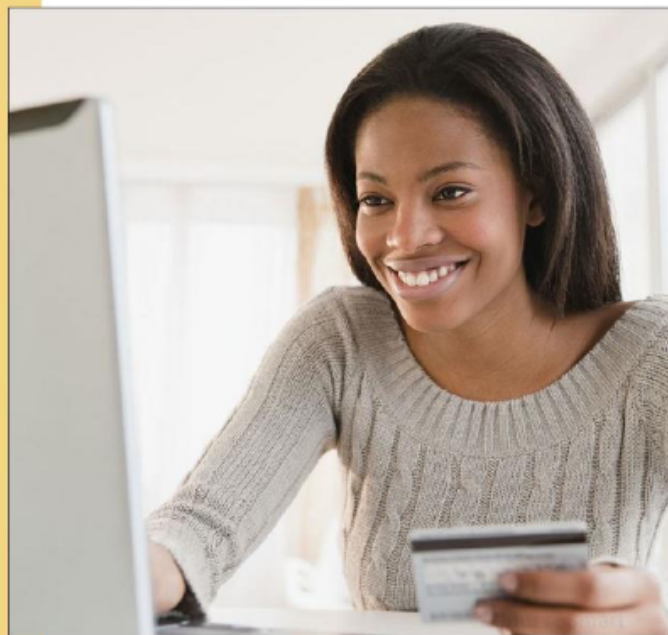
L04 Compute the variance and standard deviation of a probability distribution.

L05 Describe and compute probabilities for a binomial distribution.

L06 Describe and compute probabilities for a hypergeometric distribution.

L07 Describe and compute probabilities for a Poisson distribution.

Discrete Probability Distributions



Recent statistics suggest that 15 percent of those who visit a retail site on the Web make a purchase. A retailer wished to verify this claim. To do so, she selected a sample of 16 "hits" to her site and found that 4 had actually made a purchase. What is the likelihood of exactly four purchases? How many purchases should she expect? What is the likelihood that four or more "hits" result in a purchase? (See Exercise 49 and L05.)





6.1 Introduction

Chapters 2 through 4 are devoted to descriptive statistics. We describe raw data by organizing it into a frequency distribution and portraying the distribution in tables, graphs, and charts. Also, we compute a measure of location—such as the arithmetic mean, median, or mode—to locate a typical value near the center of the distribution. The range and the standard deviation are used to describe the spread in the data. These chapters focus on describing *something that has already happened*.

Starting with Chapter 5, the emphasis changes—we begin examining *something that would probably happen*. We note that this facet of statistics is called *statistical inference*. The objective is to make inferences (statements) about a population based on a number of observations, called a sample, selected from the population. In Chapter 5, we state that a probability is a value between 0 and 1 inclusive, and we examine how probabilities can be combined using rules of addition and multiplication.

This chapter will begin the study of **probability distributions**. A probability distribution gives the entire range of values that can occur based on an experiment. A probability distribution is similar to a relative frequency distribution. However, instead of describing the past, it describes a likely future event. For example, a drug manufacturer may claim a treatment will cause weight loss for 80 percent of the population. A consumer protection agency may test the treatment on a sample of six people. If the manufacturer's claim is true, it is *almost impossible* to have an outcome where no one in the sample loses weight and it is *most likely* that five out of the six do lose weight.

In this chapter, we discuss the mean, variance, and standard deviation of a probability distribution. We also discuss three frequently occurring probability distributions: the binomial, hypergeometric, and Poisson.

6.2 What Is a Probability Distribution?

A probability distribution shows the possible outcomes of an experiment and the probability of each of these outcomes.

LO1 Identify the characteristics of a probability distribution.

PROBABILITY DISTRIBUTION A listing of all the outcomes of an experiment and the probability associated with each outcome.

Below are the major characteristics of a probability distribution.

CHARACTERISTICS OF A PROBABILITY DISTRIBUTION

1. The probability of a particular outcome is between 0 and 1 inclusive.
2. The outcomes are mutually exclusive events.
3. The list is exhaustive. So the sum of the probabilities of the various events is equal to 1.

How can we generate a probability distribution? The following example will explain.

Example

Suppose we are interested in the number of heads showing face up on three tosses of a coin. This is the experiment. The possible results are: zero heads, one head, two heads, and three heads. What is the probability distribution for the number of heads?

Solution

There are eight possible outcomes. A tail might appear face up on the first toss, another tail on the second toss, and another tail on the third toss of the coin. Or we might get a tail, tail, and head, in that order. We use the multiplication formula for counting outcomes (5–8). There are $(2)(2)(2)$ or 8 possible results. These results are on the next page.





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Possible Result	Coin Toss			Number of Heads
	First	Second	Third	
1	T	T	T	0
2	T	T	H	1
3	T	H	T	1
4	T	H	H	2
5	H	T	T	1
6	H	T	H	2
7	H	H	T	2
8	H	H	H	3

Note that the outcome "zero heads" occurred only once, "one head" occurred three times, "two heads" occurred three times, and the outcome "three heads" occurred only once. That is, "zero heads" happened one out of eight times. Thus, the probability of zero heads is one-eighth, the probability of one head is three-eighths, and so on. The probability distribution is shown in Table 6-1. Because one of these outcomes must happen, the total of the probabilities of all possible events is 1.000. This is always true. The same information is shown in Chart 6-1.

TABLE 6-1 Probability Distribution for the Events of Zero, One, Two, and Three Heads Showing Face Up on Three Tosses of a Coin

Number of Heads, x	Probability of Outcome, $P(x)$
0	$\frac{1}{8} = .125$
1	$\frac{3}{8} = .375$
2	$\frac{3}{8} = .375$
3	$\frac{1}{8} = .125$
Total	$\frac{8}{8} = 1.000$

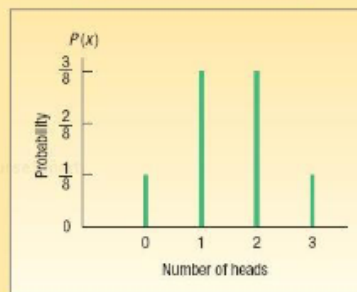


CHART 6-1 Graphical Presentation of the Number of Heads Resulting from Three Tosses of a Coin and the Corresponding Probability





Refer to the coin-tossing example in Table 6-1. We write the probability of x as $P(x)$. So the probability of zero heads is $P(0 \text{ heads}) = .125$, and the probability of one head is $P(1 \text{ head}) = .375$, and so forth. The sum of these mutually exclusive probabilities is 1; that is, from Table 6-1, $0.125 + 0.375 + 0.375 + 0.125 = 1.00$.

Self-Review 6-1

- The possible outcomes of an experiment involving the roll of a six-sided die are a one-spot, a two-spot, a three-spot, a four-spot, a five-spot, and a six-spot.
- Develop a probability distribution for the number of possible spots.
 - Portray the probability distribution graphically.
 - What is the sum of the probabilities?

6.3 Random Variables

In any experiment of chance, the outcomes occur randomly. So it is often called a *random variable*. For example, rolling a single die is an experiment: any one of six possible outcomes can occur. Some experiments result in outcomes that are quantitative (such as dollars, weight, or number of children), and others result in qualitative outcomes (such as color or religious preference). Each value of the random variable is associated with a probability to indicate the chance of a particular outcome. A few examples will further illustrate what is meant by a **random variable**.

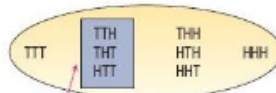
- If we count the number of employees absent from the day shift on Monday, the number might be 0, 1, 2, 3, The number absent is the random variable.
- If we weigh four steel ingots, the weights might be 2,492 pounds, 2,497 pounds, 2,506 pounds, and so on. The weight is the random variable.
- If we toss two coins and count the number of heads, there could be zero, one, or two heads. Because the number of heads resulting from this experiment is due to chance, the number of heads appearing is the random variable.
- Other random variables might be the number of defective lightbulbs produced in an hour at the Cleveland Company Inc., the grade level (9, 10, 11, or 12) of the members of the St. James girls' basketball team, the number of runners in the Boston Marathon for the 2010 race, and the daily number of drivers charged with driving under the influence of alcohol in Texas.

RANDOM VARIABLE A quantity resulting from an experiment that, by chance, can assume different values.

The following diagram illustrates the terms *experiment*, *outcome*, *event*, and *random variable*. First, for the experiment where a coin is tossed three times, there are eight possible outcomes. In this experiment, we are interested in the event that one head occurs in the three tosses. The random variable is the number of heads. In terms of probability, we want to know the probability of the event that the random variable equals 1. The result is $P(1 \text{ head in 3 tosses}) = 0.375$.

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Possible outcomes for three coin tosses



The event {one head} occurs and the random variable $x = 1$.

A random variable may be either *discrete* or *continuous*.





Discrete Random Variable

A discrete random variable can assume only a certain number of separated values. If there are 100 employees, then the count of the number absent on Monday can only be 0, 1, 2, 3, ..., 100. A discrete random variable is usually the result of counting something.

L02 Distinguish between a discrete and a continuous random variable.

DISCRETE RANDOM VARIABLE A random variable that can assume only certain clearly separated values.

A discrete random variable can, in some cases, assume fractional or decimal values. These values must be separated—that is, have distance between them. As an example, the scores awarded by judges for technical competence and artistic form in figure skating are decimal values, such as 7.2, 8.9, and 9.7. Such values are discrete because there is distance between scores of, say, 8.3 and 8.4. A score cannot be 8.34 or 8.347, for example.

Continuous Random Variable

On the other hand, if the random variable is continuous, then the distribution is a continuous probability distribution. If we measure something such as the width of a room, the height of a person, or the pressure in an automobile tire, the variable is a *continuous random variable*. It can assume one of an infinitely large number of values, within certain limitations. As examples:

- The times of commercial flights between Atlanta and Los Angeles are 4.67 hours, 5.13 hours, and so on. The random variable is the time in hours.
- Tire pressure, measured in pounds per square inch (psi), for a new Chevy Trailblazer might be 32.78 psi, 31.62 psi, 33.07 psi, and so on. In other words, any values between 28 and 35 could reasonably occur. The random variable is the tire pressure.

Logically, if we organize a set of possible values from a random variable into a probability distribution, the result is a **probability distribution**. So what is the difference between a probability distribution and a random variable? A random variable reports the particular outcome of an experiment. A probability distribution reports all the possible outcomes as well as the corresponding probability.

The tools used, as well as the probability interpretations, are different for discrete and continuous probability distributions. This chapter is limited to the discussion and interpretation of discrete distributions. In the next chapter, we discuss continuous distributions. How do you tell the difference between the two types of distributions? Usually a discrete distribution is the result of counting something, such as:

- The number of heads appearing when a coin is tossed 3 times.
- The number of students earning an A in this class.
- The number of production employees absent from the second shift today.
- The number of 30-second commercials on NBC from 8 to 11 P.M. tonight.

Continuous distributions are usually the result of some type of measurement, such as:

- The length of each song on the latest Linkin Park CD.
- The weight of each student in this class.





- The temperature outside as you are reading this book.
- The amount of money earned by each of the more than 750 players currently on Major League Baseball team rosters.

6.4 The Mean, Variance, and Standard Deviation of a Discrete Probability Distribution

In Chapter 3, we discussed measures of location and variation for a frequency distribution. The mean reports the central location of the data, and the variance describes the spread in the data. In a similar fashion, a probability distribution is summarized by its mean and variance. We identify the mean of a probability distribution by the lowercase Greek letter mu (μ) and the standard deviation by the lowercase Greek letter sigma (σ).

Mean

The mean is a typical value used to represent the central location of a probability distribution. It also is the long-run average value of the random variable. The mean of a probability distribution is also referred to as its **expected value**. It is a weighted average where the possible values of a random variable are weighted by their corresponding probabilities of occurrence.

The mean of a discrete probability distribution is computed by the formula:

LO3 Compute the mean of a probability distribution.

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MEAN OF A PROBABILITY DISTRIBUTION

$$\mu = \sum [xP(x)]$$

[6-1]

where $P(x)$ is the probability of a particular value x . In other words, multiply each x value by its probability of occurrence, and then add these products.

Variance and Standard Deviation

As noted, the mean is a typical value used to summarize a discrete probability distribution. However, it does not describe the amount of spread (variation) in a distribution. The variance does this. The formula for the variance of a probability distribution is:

LO4 Compute the variance and standard deviation of a probability distribution.

VARIANCE OF A PROBABILITY DISTRIBUTION

$$\sigma^2 = \sum [(x - \mu)^2 P(x)]$$

[6-2]

The computational steps are

1. Subtract the mean from each value, and square this difference.
2. Multiply each squared difference by its probability.
3. Sum the resulting products to arrive at the variance.

The standard deviation, σ , is found by taking the positive square root of σ^2 ; that is, $\sigma = \sqrt{\sigma^2}$.

An example will help explain the details of the calculation and interpretation of the mean and standard deviation of a probability distribution.



**Example**

John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

Number of Cars Sold, x	Probability, $P(x)$
0	.10
1	.20
2	.30
3	.30
4	.10
Total	1.00

1. What type of distribution is this?
2. On a typical Saturday, how many cars does John expect to sell?
3. What is the variance of the distribution?

Solution

1. This is a discrete probability distribution for the random variable called "number of cars sold." Note that John expects to sell only within a certain range of cars; he does not expect to sell 5 cars or 50 cars. Further, he cannot sell half a car. He can sell only 0, 1, 2, 3, or 4 cars. Also, the outcomes are mutually exclusive—he cannot sell a total of both 3 and 4 cars on the same Saturday. The sum of the possible outcomes total 1. Hence, these circumstance qualify as a probability distribution.
2. The mean number of cars sold is computed by weighting the number of cars sold by the probability of selling that number and adding or summing the products, using formula (6-1):

$$\begin{aligned}\mu &= \sum [xP(x)] \\ &= 0(.10) + 1(.20) + 2(.30) + 3(.30) + 4(.10) \\ &= 2.1\end{aligned}$$

These calculations are summarized in the following table.

Number of Cars Sold, x	Probability, $P(x)$	$x \cdot P(x)$
0	.10	0.00
1	.20	0.20
2	.30	0.60
3	.30	0.90
4	.10	0.40
Total	1.00	$\mu = 2.10$

How do we interpret a mean of 2.1? This value indicates that, over a large number of Saturdays, John Ragsdale expects to sell a mean of 2.1 cars a day. Of course, it is not possible for him to sell *exactly* 2.1 cars on any particular Saturday. However, the expected value can be used to predict the arithmetic mean number of cars sold on Saturdays in the long run. For example,





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if John works 50 Saturdays during a year, he can expect to sell $(50)(2.1)$ or 105 cars just on Saturdays. Thus, the mean is sometimes called the expected value.

3. Again, a table is useful for systemizing the computations for the variance, which is 1.290.

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Number of Cars Sold, x	Probability, $P(x)$	$(x - \mu)$	$(x - \mu)^2$	$(x - \mu)^2 P(x)$
0	.10	$0 - 2.1$	4.41	0.441
1	.20	$1 - 2.1$	1.21	0.242
2	.30	$2 - 2.1$	0.01	0.003
3	.30	$3 - 2.1$	0.81	0.243
4	.10	$4 - 2.1$	3.61	0.361
				$\sigma^2 = 1.290$

Recall that the standard deviation, σ , is the positive square root of the variance. In this example, $\sqrt{\sigma^2} = \sqrt{1.290} = 1.136$ cars. How do we interpret a standard deviation of 1.136 cars? If salesperson Rita Kirsch also sold a mean of 2.1 cars on Saturdays, and the standard deviation in her sales was 1.91 cars, we would conclude that there is more variability in the Saturday sales of Ms. Kirsch than in those of Mr. Ragsdale (because $1.91 > 1.136$).

Self-Review 6-2



The Pizza Palace offers three sizes of cola—small, medium, and large—to go with its pizza. The colas are sold for \$0.80, \$0.90, and \$1.20, respectively. Thirty percent of the orders are for small, 50 percent are for medium, and 20 percent are for the large sizes. Organize the size of the colas and the probability of a sale into a probability distribution.

- Is this a discrete probability distribution? Indicate why or why not.
- Compute the mean amount charged for a cola.
- What is the variance in the amount charged for a cola? The standard deviation?

Exercises

connect

1. Compute the mean and variance of the following discrete probability distribution.

x	$P(x)$
0	.2
1	.4
2	.3
3	.1

2. Compute the mean and variance of the following discrete probability distribution.

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x	$P(x)$
2	.5
8	.3
10	.2





3. Compute the mean and variance of the following probability distribution.

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x	$P(x)$
5	.1
10	.3
15	.2
20	.4

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4. Which of these variables are discrete and which are continuous random variables?
- The number of new accounts established by a salesperson in a year.
 - The time between customer arrivals to a bank ATM.
 - The number of customers in Big Nick's barber shop.
 - The amount of fuel in your car's gas tank.
 - The number of minorities on a jury.
 - The outside temperature today.
5. The information below is the number of daily emergency service calls made by the volunteer ambulance service of Walterboro, South Carolina, for the last 50 days. To explain, there were 22 days on which there were 2 emergency calls, and 9 days on which there were 3 emergency calls.

Number of Calls	Frequency
0	8
1	10
2	22
3	9
4	1
Total	50

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- Convert this information on the number of calls to a probability distribution.
 - Is this an example of a discrete or continuous probability distribution?
 - What is the mean number of emergency calls per day?
 - What is the standard deviation of the number of calls made daily?
6. The director of admissions at Kinzua University in Nova Scotia estimated the distribution of student admissions for the fall semester on the basis of past experience. What is the expected number of admissions for the fall semester? Compute the variance and the standard deviation of the number of admissions.

Admissions	Probability
1,000	.6
1,200	.3
1,500	.1

7. Belk Department Store is having a special sale this weekend. Customers charging purchases of more than \$50 to their Belk credit card will be given a special Belk Lottery card. The customer will scratch off the card, which will indicate the amount to be taken off the total amount of the purchase. Listed below are the amount of the prize and the percent of the time that amount will be deducted from the total amount of the purchase.

Prize Amount	Probability
\$ 10	.50
25	.40
50	.08
100	.02

- What is the mean amount deducted from the total purchase amount?
- What is the standard deviation of the amount deducted from the total purchase?





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8. The Downtown Parking Authority of Tampa, Florida, reported the following information for a sample of 250 customers on the number of hours cars are parked and the amount they are charged.

Number of Hours	Frequency	Amount Charged
1	20	\$ 3.00
2	38	6.00
3	53	9.00
4	45	12.00
5	40	14.00
6	13	16.00
7	5	18.00
8	36	20.00
	250	

- Convert the information on the number of hours parked to a probability distribution. Is this a discrete or a continuous probability distribution?
- Find the mean and the standard deviation of the number of hours parked. How would you answer the question: How long is a typical customer parked?
- Find the mean and the standard deviation of the amount charged.

6.5 Binomial Probability Distribution

LO5 Describe and compute probabilities for a binomial distribution.

The **binomial probability distribution** is a widely occurring discrete probability distribution. One characteristic of a binomial distribution is that there are only two possible outcomes on a particular trial of an experiment. For example, the statement in a true/false question is either true or false. The outcomes are mutually exclusive, meaning that the answer to a true/false question cannot be both true and false at the same time. As other examples, a product is classified as either acceptable or not acceptable by the quality control department, a worker is classified as employed or unemployed, and a sales call results in the customer either purchasing the product or not purchasing the product. Frequently, we classify the two possible outcomes as "success" and "failure." However, this classification does not imply that one outcome is good and the other is bad.

Another characteristic of the binomial distribution is that the random variable is the result of counts. That is, we count the number of successes in the total number of trials. We flip a fair coin five times and count the number of times a head appears, we select 10 workers and count the number who are over 50 years of age, or we select 20 boxes of Kellogg's Raisin Bran and count the number that weigh more than the amount indicated on the package.

A third characteristic of a binomial distribution is that the probability of a success remains the same from one trial to another. Two examples are:

- The probability you will guess the first question of a true/false test correctly (a success) is one-half. This is the first "trial." The probability that you will guess correctly on the second question (the second trial) is also one-half, the probability of success on the third trial is one-half, and so on.
- If past experience revealed the swing bridge over the Intracoastal Waterway in Socastee was raised one out of every 20 times you approach it, then the probability is one-twentieth that it will be raised (a "success") the next time you approach it, one-twentieth the following time, and so on.

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The final characteristic of a binomial probability distribution is that each trial is *independent* of any other trial. Independent means that there is no pattern to the trials. The outcome of a particular trial does not affect the outcome of any other trial. Two examples are:

- A young family has two children, both boys. The probability of a third birth being a boy is still .50. That is, the gender of the third child is independent of the other two.
- Suppose 20 percent of the patients served in the emergency room at Waccamaw Hospital do not have insurance. If the second patient served on the afternoon shift today did not have insurance, that does not affect the probability the third, the tenth, or any of the other patients will or will not have insurance.

BINOMIAL PROBABILITY EXPERIMENT

1. An outcome on each trial of an experiment is classified into one of two mutually exclusive categories—a success or a failure.
2. The random variable counts the number of successes in a fixed number of trials.
3. The probability of success and failure stay the same for each trial.
4. The trials are independent, meaning that the outcome of one trial does not affect the outcome of any other trial.

How Is a Binomial Probability Computed?

To construct a particular binomial probability, we use (1) the number of trials and (2) the probability of success on each trial. For example, if an examination at the conclusion of a management seminar consists of 20 multiple-choice questions, the number of trials is 20. If each question has five choices and only one choice is correct, the probability of success on each trial is .20. Thus, the probability is .20 that a person with no knowledge of the subject matter will guess the answer to a question correctly. So the conditions of the binomial distribution just noted are met.

A binomial probability is computed by the formula:

BINOMIAL PROBABILITY FORMULA

$$P(x) = {}_nC_x \pi^x (1 - \pi)^{n-x} \quad [6-3]$$

where:

C denotes a combination.

n is the number of trials.

x is the random variable defined as the number of successes.

π is the probability of a success on each trial.

We use the Greek letter π (pi) to denote a binomial population parameter. Do not confuse it with the mathematical constant 3.1416.

Example

There are five flights daily from Pittsburgh via US Airways into the Bradford Regional Airport in Bradford, Pennsylvania. Suppose the probability that any flight arrives late is .20. What is the probability that none of the flights are late today? What is the probability that exactly one of the flights is late today?

Solution

We can use Formula (6-3). The probability that a particular flight is late is .20, so let $\pi = .20$. There are five flights, so $n = 5$, and x , the random variable, refers to





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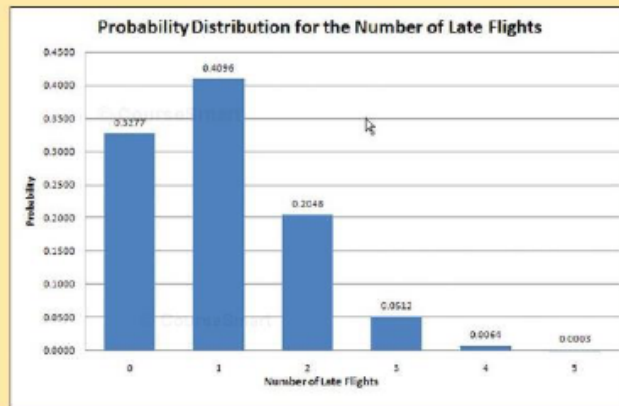
the number of successes. In this case, a "success" is a flight that arrives late. Because there are no late arrivals, $x = 0$.

$$\begin{aligned} P(0) &= {}_n C_x (\pi)^x (1 - \pi)^{n-x} \\ &= {}_5 C_0 (.20)^0 (1 - .20)^{5-0} = (1)(1)(.3277) = .3277 \end{aligned}$$

The probability that exactly one of the five flights will arrive late today is .4096, found by

$$\begin{aligned} P(1) &= {}_n C_x (\pi)^x (1 - \pi)^{n-x} \\ &= {}_5 C_1 (.20)^1 (1 - .20)^{5-1} = (5)(.20)(.4096) = .4096 \end{aligned}$$

The entire binomial probability distribution with $\pi = .20$ and $n = 5$ is shown in the following bar chart. We can observe that the probability of exactly 3 late flights is .0512 and from the bar chart that the distribution of the number of late arrivals is positively skewed.



The mean (μ) and the variance (σ^2) of a binomial distribution can be computed in a "shortcut" fashion by:

MEAN OF A BINOMIAL DISTRIBUTION

$$\mu = n\pi$$

[6-4]

VARIANCE OF A BINOMIAL DISTRIBUTION

$$\sigma^2 = n\pi(1 - \pi)$$

[6-5]

For the example regarding the number of late flights, recall that $\pi = .20$ and $n = 5$. Hence:

$$\mu = n\pi = (5)(.20) = 1.0$$

$$\sigma^2 = n\pi(1 - \pi) = 5(.20)(1 - .20) = .80$$





The mean of 1.0 and the variance of .80 can be verified from formulas (6-1) and (6-2). The probability distribution from the Excel output on the previous page and the details of the calculations are shown below.

Number of Late Flights, x	$P(x)$	$xP(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2P(x)$
0	0.3277	0.0000	-1	1	0.3277
1	0.4096	0.4096	0	0	0
2	0.2048	0.4096	1	1	0.2048
3	0.0512	0.1536	2	4	0.2048
4	0.0064	0.0256	3	9	0.0576
5	0.0003	0.0015	4	16	0.0048
			$\mu = 1.0000$		$\sigma^2 = 0.7997$

Binomial Probability Tables

Formula (6-3) can be used to build a binomial probability distribution for any value of n and π . However, for a larger n , the calculations take more time. For convenience, the tables in Appendix B.9 show the result of using the formula for various values of n and π . Table 6-2 shows part of Appendix B.9 for $n = 6$ and various values of π .

TABLE 6-2 Binomial Probabilities for $n = 6$ and Selected Values of π

		$n = 6$ Probability									
$x \backslash \pi$.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95
0	.735	.531	.262	.118	.047	.016	.004	.001	.000	.000	.000
1	.232	.354	.393	.303	.187	.094	.037	.010	.002	.000	.000
2	.031	.098	.246	.324	.311	.234	.138	.060	.015	.001	.000
3	.002	.015	.082	.185	.276	.313	.276	.185	.082	.015	.002
4	.000	.001	.015	.060	.138	.234	.311	.324	.246	.098	.031
5	.000	.000	.002	.010	.037	.094	.187	.303	.393	.354	.232
6	.000	.000	.000	.001	.004	.016	.047	.118	.262	.531	.735

Example

Five percent of the worm gears produced by an automatic, high-speed Carter-Bell milling machine are defective. What is the probability that out of six gears selected at random none will be defective? Exactly one? Exactly two? Exactly three? Exactly four? Exactly five? Exactly six out of six?

Solution

The binomial conditions are met: (a) there are only two possible outcomes (a particular gear is either defective or acceptable), (b) there is a fixed number of trials (6), (c) there is a constant probability of success (.05), and (d) the trials are independent.

Refer to Table 6-2 above for the probability of exactly zero defective gears. Go down the left margin to an x of 0. Now move horizontally to the column headed by a π of .05 to find the probability. It is .735.

The probability of exactly one defective in a sample of six worm gears is .232. The complete binomial probability distribution for $n = 6$ and $\pi = .05$ is:





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Number of Defective Gears, x	Probability of Occurrence, $P(x)$	Number of Defective Gears, x	Probability of Occurrence, $P(x)$
0	.735	4	.000
1	.232	5	.000
2	.031	6	.000
3	.002		

Of course, there is a slight chance of getting exactly five defective gears out of six random selections. It is .00000178, found by inserting the appropriate values in the binomial formula:

$$P(5) = {}_6C_5(.05)^5(.95)^1 = (6)(.05)^5(.95) = .00000178$$

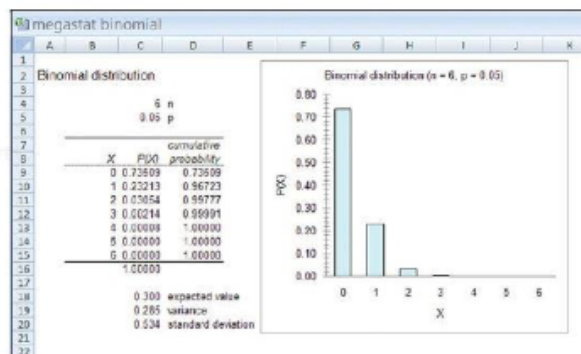
For six out of the six, the exact probability is .00000016. Thus, the probability is very small that five or six defective gears will be selected in a sample of six.

We can compute the mean or expected value of the distribution of the number defective:

$$\mu = n\pi = (6)(.05) = 0.30$$

$$\sigma^2 = n\pi(1 - \pi) = 6(.05)(.95) = 0.285$$

MegaStat software will also compute the probabilities for a binomial distribution. Below is the output for the previous example. In MegaStat, p is used to represent the probability of success rather than π . The cumulative probability, expected value, variance, and standard deviation are also reported.



Self-Review 6-3



Eighty percent of the employees at the J. M. Smucker Company plant on Laskey Road have their bimonthly wages sent directly to their bank by electronic funds transfer. This is also called direct deposit. Suppose we select a random sample of seven employees.

- Does this situation fit the assumptions of the binomial distribution?
- What is the probability that all seven employees use direct deposit?
- Use formula (6-3) to determine the exact probability that four of the seven sampled employees use direct deposit.
- Use Appendix B.9 to verify your answers to parts (b) and (c).





	A	B
1	Success	Probability
2	0	0.0230
3	1	0.0910
4	2	0.1754
5	3	0.2198
6	4	0.2011
7	5	0.1492
8	6	0.0826
9	7	0.0397
10	8	0.0162
11	9	0.0057
12	10	0.0017
13	11	0.0005
14	12	0.0001
15	13	0.0000
16	14	0.0000
17	15	0.0000

Appendix B.9 is limited. It gives probabilities for n values from 1 to 15 and π values of .05, .10, . . . , .90, and .95. A software program can generate the probabilities for a specified number of successes, given n and π . The Excel output to the left shows the probability when $n = 40$ and $\pi = .09$. Note that the number of successes stops at 15 because the probabilities for 16 to 40 are very close to 0. The instructions are detailed in the Software Commands section on page 219.

Several additional points should be made regarding the binomial probability distribution.

1. If n remains the same but π increases from .05 to .95, the shape of the distribution changes. Look at Table 6-3 and Chart 6-2. The distribution for a π of .05 is positively skewed. As π approaches .50, the distribution becomes symmetrical. As π goes beyond .50 and moves toward .95, the probability distribution becomes negatively skewed. Table 6-3 highlights probabilities for $n = 10$ and π of .05, .10, .20, .50, and .70. The graphs of these probability distributions are shown in Chart 6-2.

TABLE 6-3 Probability of 0, 1, 2, . . . Successes for a π of .05, .10, .20, .50, and .70 and an n of 10

$x \backslash \pi$.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95
0	.599	.349	.107	.028	.006	.001	.000	.000	.000	.000	.000
1	.315	.387	.268	.121	.040	.010	.002	.000	.000	.000	.000
2	.075	.194	.302	.233	.121	.044	.011	.001	.000	.000	.000
3	.010	.057	.201	.267	.215	.117	.042	.009	.001	.000	.000
4	.001	.011	.088	.200	.251	.205	.111	.037	.006	.000	.000
5	.000	.001	.026	.103	.201	.246	.201	.103	.026	.001	.000
6	.000	.000	.006	.037	.111	.205	.251	.200	.088	.011	.001
7	.000	.000	.001	.009	.042	.117	.215	.267	.201	.057	.010
8	.000	.000	.000	.001	.011	.044	.121	.233	.302	.194	.075
9	.000	.000	.000	.000	.002	.010	.040	.121	.268	.387	.315
10	.000	.000	.000	.000	.000	.001	.006	.028	.107	.349	.599

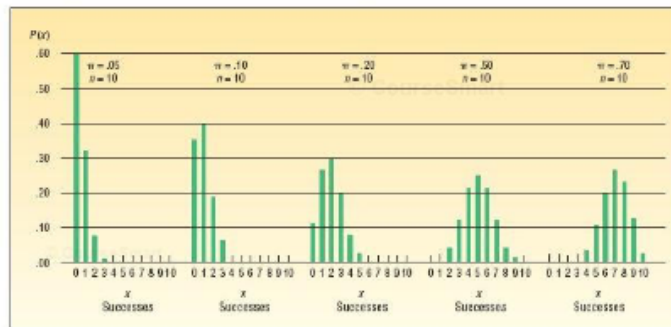


CHART 6-2 Graphing the Binomial Probability Distribution for a π of .05, .10, .20, .50, and .70 and an n of 10

2. If π , the probability of success, remains the same but n becomes larger, the shape of the binomial distribution becomes more symmetrical. Chart 6-3 shows a situation where π remains constant at .10 but n increases from 7 to 40.





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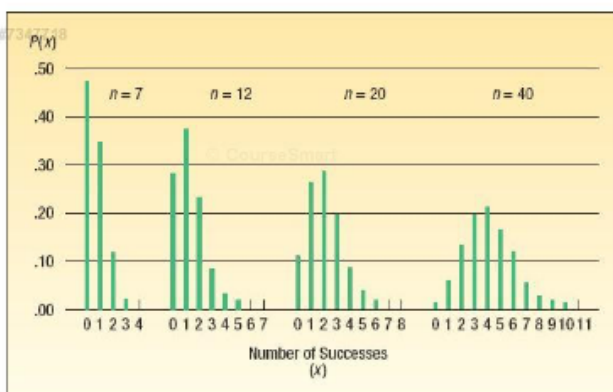


CHART 6-3 Chart Representing the Binomial Probability Distribution for a π of .10 and an n of 7, 12, 20, and 40

Exercises

connect

9. In a binomial situation, $n = 4$ and $\pi = .25$. Determine the probabilities of the following events using the binomial formula.
 - a. $x = 2$
 - b. $x = 3$
10. In a binomial situation, $n = 5$ and $\pi = .40$. Determine the probabilities of the following events using the binomial formula.
 - a. $x = 1$
 - b. $x = 2$
11. Assume a binomial distribution where $n = 3$ and $\pi = .60$.
 - a. Refer to Appendix B.9, and list the probabilities for values of x from 0 to 3.
 - b. Determine the mean and standard deviation of the distribution from the general definitions given in formulas (6-1) and (6-2).
12. Assume a binomial distribution where $n = 5$ and $\pi = .30$.
 - a. Refer to Appendix B.9, and list the probabilities for values of x from 0 to 5.
 - b. Determine the mean and standard deviation of the distribution from the general definitions given in formulas (6-1) and (6-2).
13. An American Society of Investors survey found 30 percent of individual investors have used a discount broker. In a random sample of nine individuals, what is the probability:
 - a. Exactly two of the sampled individuals have used a discount broker?
 - b. Exactly four of them have used a discount broker?
 - c. None of them have used a discount broker?
14. The United States Postal Service reports 95 percent of first class mail within the same city is delivered within two days of the time of mailing. Six letters are randomly sent to different locations.
 - a. What is the probability that all six arrive within two days?
 - b. What is the probability that exactly five arrive within two days?
 - c. Find the mean number of letters that will arrive within two days.
 - d. Compute the variance and standard deviation of the number that will arrive within two days.
15. Industry standards suggest that 10 percent of new vehicles require warranty service within the first year. Jones Nissan in Sumter, South Carolina, sold 12 Nissans yesterday.
 - a. What is the probability that none of these vehicles requires warranty service?
 - b. What is the probability exactly one of these vehicles requires warranty service?





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- c. Determine the probability that exactly two of these vehicles require warranty service.
d. Compute the mean and standard deviation of this probability distribution.
16. A telemarketer makes six phone calls per hour and is able to make a sale on 30 percent of these contacts. During the next two hours, find:
- The probability of making exactly four sales.
 - The probability of making no sales.
 - The probability of making exactly two sales.
 - The mean number of sales in the two-hour period.
17. A recent survey by the American Accounting Association revealed 23 percent of students graduating with a major in accounting select public accounting. Suppose we select a sample of 15 recent graduates.
- What is the probability two select public accounting?
 - What is the probability five select public accounting?
 - How many graduates would you expect to select public accounting?
18. It is reported that 16 percent of American households use a cell phone exclusively for their telephone service. In a sample of eight households, find the probability that:
- None use a cell phone as their exclusive service.
 - At least one uses the cell exclusively.
 - At least five use the cell phone.

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Cumulative Binomial Probability Distributions

We may wish to know the probability of correctly guessing the answers to 6 or more true/false questions out of 10. Or we may be interested in the probability of selecting less than two defectives at random from production during the previous hour. In these cases, we need cumulative frequency distributions similar to the ones developed in Chapter 2. See page 42. The following example will illustrate.

Example

A study by the Illinois Department of Transportation concluded that 76.2 percent of front seat occupants used seat belts. That means that both occupants of the front seat were using their seat belts. Suppose we decide to compare that information with current usage. We select a sample of 12 vehicles.

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- What is the probability the front seat occupants in exactly 7 of the 12 vehicles selected are wearing seat belts?
- What is the probability the front seat occupants in at least 7 of the 12 vehicles are wearing seat belts?

Solution

This situation meets the binomial requirements.

- In a particular vehicle, both the front seat occupants are either wearing seat belts or they are not. There are only two possible outcomes.
- There are a fixed number of trials, 12 in this case, because 12 vehicles are checked.
- The probability of a "success" (occupants wearing seat belts) is the same from one vehicle to the next: 76.2 percent.
- The trials are independent. If the fourth vehicle selected in the sample has all the occupants wearing their seat belts, this does not have any effect on the results for the fifth or tenth vehicle.

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To find the likelihood the occupants of exactly 7 of the sampled vehicles are wearing seat belts, we use formula 6-3. In this case, $n = 12$ and $\pi = .762$.

$$P(x = 7 | n = 12 \text{ and } \pi = .762) = {}_{12}C_7(.762)^7(1 - .762)^{12-7} = 792(.149171)(.000764) = .0902$$

So we conclude the likelihood that the occupants of exactly 7 of the 12 sampled vehicles will be wearing their seat belts is about 9 percent. We often use, as we





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did in this equation, a bar “|” to mean “given that.” So in this equation we want to know the probability that x is equal to 7 “given that the number of trials is 12 and the probability of a success is .762.”

To find the probability that the occupants in 7 or more of the vehicles will be wearing seat belts, we use formula (6-3) from this chapter as well as the special rule of addition from the previous chapter. See formula (5-2) on page 153.

Because the events are mutually exclusive (meaning that a particular sample of 12 vehicles cannot have both a *total* of 7 and a *total* of 8 vehicles where the occupants are wearing seat belts), we find the probability of 7 vehicles where the occupants are wearing seat belts, the probability of 8, and so on up to the probability that occupants of all 12 sample vehicles are wearing seat belts. The probability of each of these outcomes is then totaled.

$$\begin{aligned}
 P(x \geq 7 | n = 12 \text{ and } \pi = .762) \\
 &= P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12) \\
 &= .0902 + .1805 + .2569 + .2467 + .1436 + .0383 \\
 &= .9562
 \end{aligned}$$

So the probability of selecting 12 cars and finding that the occupants of 7 or more vehicles were wearing seat belts is .9562. This information is shown on the following Excel spreadsheet. There is a slight difference in the software answer due to rounding. The Excel commands are similar to those detailed in the Software Commands section on page 219, number 2.

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Cumulative binomial					
	A	B	C	D	E
1	Success	Probability			
2	0	0.0000			
3	1	0.0000			
4	2	0.0000			
5	3	0.0002			
6	4	0.0017			
7	5	0.0089			
8	6	0.0329			
9	7	0.0902			
10	8	0.1805			
11	9	0.2569			
12	10	0.2467			
13	11	0.1436			
14	12	0.0383			
15		0.9563			

Sum of Probabilities
for 7 or more
successes

Self-Review 6-4



For a case where $n = 4$ and $\pi = .60$, determine the probability that:

- $x = 2$.
- $x \leq 2$.
- $x > 2$.

Exercises

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19. In a binomial distribution, $n = 8$ and $\pi = .30$. Find the probabilities of the following events.

- $x = 2$.
- $x \leq 2$ (the probability that x is equal to or less than 2).
- $x \geq 3$ (the probability that x is equal to or greater than 3).

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20. In a binomial distribution, $n = 12$ and $\pi = .60$. Find the following probabilities.
- $X = 5$.
 - $X \leq 5$.
 - $X \geq 6$.
21. In a recent study, 90 percent of the homes in the United States were found to have large-screen TVs. In a sample of nine homes, what is the probability that:
- All nine have large-screen TVs?
 - Less than five have large-screen TVs?
 - More than five have large-screen TVs?
 - At least seven homes have large-screen TVs?
22. A manufacturer of window frames knows from long experience that 5 percent of the production will have some type of minor defect that will require an adjustment. What is the probability that in a sample of 20 window frames:
- None will need adjustment?
 - At least one will need adjustment?
 - More than two will need adjustment?
23. The speed with which utility companies can resolve problems is very important. GTC, the Georgetown Telephone Company, reports it can resolve customer problems the same day they are reported in 70 percent of the cases. Suppose the 15 cases reported today are representative of all complaints.
- How many of the problems would you expect to be resolved today? What is the standard deviation?
 - What is the probability 10 of the problems can be resolved today?
 - What is the probability 10 or 11 of the problems can be resolved today?
 - What is the probability more than 10 of the problems can be resolved today?
24. It is asserted that 80 percent of the cars approaching an individual toll both in New Jersey are equipped with an E-ZPass transponder. Find the probability that in a sample of six cars:
- All six will have the transponder.
 - At least three will have the transponder.
 - None will have a transponder.

6.6 Hypergeometric Probability Distribution

For the binomial distribution to be applied, the probability of a success must stay the same for each trial. For example, the probability of guessing the correct answer to a true/false question is .50. This probability remains the same for each question on an examination. Likewise, suppose that 40 percent of the registered voters in a precinct are Republicans. If 27 registered voters are selected at random, the probability of choosing a Republican on the first selection is .40. The chance of choosing a Republican on the next selection is also .40, assuming that the sampling is done *with replacement*, meaning that the person selected is put back in the population before the next person is selected.

Most sampling, however, is done *without replacement*. Thus, if the population is small, the probability for each observation will change. For example, if the population consists of 20 items, the probability of selecting a particular item from that population is 1/20. If the sampling is done without replacement, after the first selection there are only 19 items remaining; the probability of selecting a particular item on the second selection is only 1/19. For the third selection, the probability is 1/18, and so on. This assumes that the population is **finite**—that is, the number in the population is known and relatively small in number. Examples of a finite population are 2,842 Republicans in the precinct, 9,241 applications for medical school, and the 18 2010 Dakota 4x4 Crew Cabs at Helfman Dodge Chrysler Jeep in Houston, TX.

Recall that one of the criteria for the binomial distribution is that the probability of success remains the same from trial to trial. Because the probability of success does not remain the same from trial to trial when sampling is from a relatively small population without replacement, the binomial distribution should not be used. Instead, the **hypergeometric distribution** is applied. Therefore, (1) if a sample is

L06 Describe and compute probabilities for a hypergeometric distribution.





selected from a finite population without replacement and (2) if the size of the sample n is more than 5 percent of the size of the population N , then the hypergeometric distribution is used to determine the probability of a specified number of successes or failures. It is especially appropriate when the size of the population is small. The formula for the hypergeometric distribution is:

HYPERGEOMETRIC DISTRIBUTION

$$P(x) = \frac{{}_s C_x {}_{(N-s)} C_{n-x}}{N C_n}$$

[6-6]

where:

N is the size of the population.

S is the number of successes in the population.

x is the number of successes in the sample. It may be 0, 1, 2, 3, . . .

n is the size of the sample or the number of trials.

C is the symbol for a combination.

In summary, a hypergeometric probability distribution has these characteristics:

HYPERGEOMETRIC PROBABILITY EXPERIMENT

1. An outcome on each trial of an experiment is classified into one of two mutually exclusive categories—a success or a failure.
2. The random variable is the number of successes in a fixed number of trials.
3. The trials are *not independent*.
4. We assume that we sample from a finite population without replacement and $n/N > 0.05$. So, the probability of a success *changes* for each trial.

The following example illustrates the details of determining a probability using the hypergeometric distribution.

Example

PlayTime Toys Inc. employs 50 people in the Assembly Department. Forty of the employees belong to a union and ten do not. Five employees are selected at random to form a committee to meet with management regarding shift starting times. What is the probability that four of the five selected for the committee belong to a union?

**Solution**

The population in this case is the 50 Assembly Department employees. An employee can be selected for the committee only once. Hence, the sampling is done without replacement. Thus, the probability of selecting a union employee, for example, changes from one trial to the next. The hypergeometric distribution is appropriate for determining the probability. In this problem,

N is 50, the number of employees.

S is 40, the number of union employees.

x is 4, the number of union employees selected.

n is 5, the number of employees selected.





We wish to find the probability 4 of the 5 committee members belong to a union. Inserting these values into formula (6-6):

$$P(4) = \frac{{}_{40}C_4({}_{50-40}C_{5-4})}{{}_{50}C_5} = \frac{(91,390)(10)}{2,118,760} = .431$$

Thus, the probability of selecting 5 assembly workers at random from the 50 workers and finding 4 of the 5 are union members is .431.

Table 6-4 shows the hypergeometric probabilities of finding 0, 1, 2, 3, 4, and 5 union members on the committee.

TABLE 6-4 Hypergeometric Probabilities ($n = 5$, $N = 50$, and $S = 40$) for the Number of Union Members on the Committee

Union Members	Probability
0	.000
1	.004
2	.044
3	.210
4	.431
5	.311
	1.000

In order for you to compare the two probability distributions, Table 6-5 shows the hypergeometric and binomial probabilities for the PlayTime Toys Inc. example. Because 40 of the 50 Assembly Department employees belong to the union, we let $\pi = .80$ for the binomial distribution. The binomial probabilities for Table 6-5 come from the binomial distribution with $n = 5$ and $\pi = .80$.

TABLE 6-5 Hypergeometric and Binomial Probabilities for PlayTime Toys Inc. Assembly Department

Number of Union Members on Committee	Hypergeometric Probability, $P(x)$	Binomial Probability ($n = 5$ and $\pi = .80$)
0	.000	.000
1	.004	.006
2	.044	.051
3	.210	.205
4	.431	.410
5	.311	.328
	1.000	1.000

When the binomial requirement of a constant probability of success cannot be met, the hypergeometric distribution should be used. However, as Table 6-5 shows, under certain conditions the results of the binomial distribution can be used to approximate the hypergeometric. This leads to a rule of thumb:

If selected items are not returned to the population, the binomial distribution can be used to closely approximate the hypergeometric distribution when $n < .05N$. In words, the binomial will suffice if the sample is less than 5 percent of the population.

A hypergeometric distribution can be created using Excel. See the output on the left. The necessary steps are given in the Software Commands section on page 219 at the end of the chapter.

Union Members	Probability
0	0.000
1	0.004
2	0.044
3	0.210
4	0.431
5	0.311





Self-Review 6-5 Horwege Discount Brokers plans to hire 5 new financial analysts this year. There is a pool of 12 approved applicants, and George Horwege, the owner, decides to randomly select those who will be hired. There are 6 men and 4 women among the approved applicants. What is the probability that 3 of the 5 hired are men?



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Exercises

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25. A CD contains 10 songs; 6 are classical and 4 are rock and roll. In a sample of 3 songs, what is the probability that exactly 2 are classical? Assume the samples are drawn without replacement.
26. A population consists of 15 items, 10 of which are acceptable. In a sample of 4 items, what is the probability that exactly 3 are acceptable? Assume the samples are drawn without replacement.
27. Kolzak Appliance Outlet just received a shipment of 10 DVD players. Shortly after they were received, the manufacturer called to report that he had inadvertently shipped 3 defective units. Ms. Kolzak, the owner of the outlet, decided to test 2 of the 10 DVD players she received. What is the probability that neither of the 2 DVD players tested is defective? Assume the samples are drawn without replacement.
28. The Computer Systems Department has 6 faculty, 6 of whom are tenured. Dr. Vonder, the chairman, wants to establish a committee of 3 department faculty members to review the curriculum. If she selects the committee at random:
 - a. What is the probability all members of the committee are tenured?
 - b. What is the probability that at least one member is not tenured? (Hint: For this question, use the complement rule.)
29. Keith's Florists has 15 delivery trucks, used mainly to deliver flowers and flower arrangements in the Greenville, South Carolina, area. Of these 15 trucks, 6 have brake problems. A sample of 5 trucks is randomly selected. What is the probability that 2 of those tested have defective brakes?
30. The game called Lotto sponsored by the Louisiana Lottery Commission pays its largest prize when a contestant matches all 6 of the 40 possible numbers. Assume there are 40 ping-pong balls each with a single number between 1 and 40. Any number appears only once, and the winning balls are selected without replacement.
 - a. The commission reports that the probability of matching all the numbers are 1 in 3,838,380. What is this in terms of probability?
 - b. Use the hypergeometric formula to find this probability.

The lottery commission also pays if a contestant matches 4 or 5 of the 6 winning numbers. Hint: Divide the 40 numbers into two groups, winning numbers and nonwinning numbers.

 - c. Find the probability, again using the hypergeometric formula, for matching 4 of the 6 winning numbers.
 - d. Find the probability of matching 5 of the 6 winning numbers.

L07 Describe and compute probabilities for a Poisson distribution.

6.7 Poisson Probability Distribution

The **Poisson probability distribution** describes the number of times some event occurs during a specified interval. The interval may be time, distance, area, or volume.

The distribution is based on two assumptions. The first assumption is that the probability is proportional to the length of the interval. The second assumption is that the intervals are independent. To put it another way, the longer the interval, the larger the probability, and the number of occurrences in one interval does not affect the other intervals. This distribution is also a limiting form of the binomial distribution when the probability of a success is very small and n is large. It is often referred to as the "law of improbable events," meaning that the probability, π , of a particular



**Statistics in Action**

Near the end of World War II, the Germans developed rocket bombs, which were fired at the city of London. The Allied military command didn't know whether these bombs were fired at random or whether they had an aiming device. To investigate, the city of London was divided into 586 square regions. The distribution of hits in each square was recorded as follows:

Hits	0	1	2	3	4	5
Regions	229	221	95	35	7	1

To interpret, the above chart indicates that 229 regions were not hit with one of the bombs. Seven regions were hit four times. Using the Poisson distribution, with a mean of 0.93 hits per region, the expected number of hits is as follows:

Hits	0	1	2	3	4	5 or more
Regions	234.2	215.0	300.0	91.0	7.2	1.8

Because the actual number of hits was close to the expected number of hits, the military command concluded that the

(continued)

event's happening is quite small. The Poisson distribution is a discrete probability distribution because it is formed by counting.

In summary, a Poisson probability distribution has these characteristics:

POISSON PROBABILITY EXPERIMENT

1. The random variable is the number of times some event occurs during a defined interval.
2. The probability of the event is proportional to the size of the interval.
3. The intervals do not overlap and are independent.

This distribution has many applications. It is used as a model to describe the distribution of errors in data entry, the number of scratches and other imperfections in newly painted car panels, the number of defective parts in outgoing shipments, the number of customers waiting to be served at a restaurant or waiting to get into an attraction at Disney World, and the number of accidents on I-75 during a three-month period.

The Poisson distribution can be described mathematically by the formula:

POISSON DISTRIBUTION

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

[6-7]

where:

μ (μ) is the mean number of occurrences (successes) in a particular interval.
 e is the constant 2.71828 (base of the Napierian logarithmic system).

x is the number of occurrences (successes).

$P(x)$ is the probability for a specified value of x .

The mean number of successes, μ , can be determined by $n\pi$, where n is the total number of trials and π the probability of success.

MEAN OF A POISSON DISTRIBUTION

$$\mu = n\pi$$

[6-8]

The variance of the Poisson is also equal to its mean. If, for example, the probability that a check cashed by a bank will bounce is .0003, and 10,000 checks are cashed, the mean and the variance for the number of bad checks is 3.0, found by $\mu = n\pi = 10,000(.0003) = 3.0$.

Recall that for a binomial distribution there is a fixed number of trials. For example, for a four-question multiple-choice test there can only be zero, one, two, three, or four successes (correct answers). The random variable, x , for a Poisson distribution, however, can assume an infinite number of values—that is, 0, 1, 2, 3, 4, 5, . . . However, the probabilities become very small after the first few occurrences (successes).

To illustrate the Poisson probability computation, assume baggage is rarely lost by Delta Airlines. Most flights do not experience any mishandled bags; some have one bag lost; a few have two bags lost; rarely a flight will have three lost bags; and so on. Suppose a random sample of 1,000 flights shows a total of 300 bags were lost. Thus, the arithmetic mean number of lost bags per flight is 0.3, found by $300/1,000$. If the number of lost bags per flight follows a Poisson distribution with $\mu = 0.3$, we can compute the various probabilities using formula (6-7):

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

For example, the probability of not losing any bags is:

$$P(0) = \frac{(0.3)^0 (e^{-0.3})}{0!} = 0.7408$$





bombs were falling at random. The Germans had not developed a bomb with an aiming device.

In other words, 74 percent of the flights will have no lost baggage. The probability of exactly one lost bag is:

$$P(1) = \frac{(0.3)^1(e^{-0.3})}{1!} = 0.2222$$

Thus, we would expect to find exactly one lost bag on 22 percent of the flights. Poisson probabilities can also be found in the table in Appendix B.5.

Example

Recall from the previous illustration that the number of lost bags follows a Poisson distribution with a mean of 0.3. Use Appendix B.5 to find the probability that no bags will be lost on a particular flight. What is the probability exactly one bag will be lost on a particular flight? When should the supervisor become suspicious that a flight is having too many lost bags?

Solution

Part of Appendix B.5 is repeated as Table 6-6. To find the probability of no lost bags, locate the column headed "0.3" and read down that column to the row labeled "0." The probability is .7408. That is the probability of no lost bags. The probability of one lost bag is .2222, which is in the next row of the table, in the same column. The probability of two lost bags is .0333, in the row below; for three lost bags, it is .0033; and for four lost bags, it is .0003. Thus, a supervisor should not be surprised to find one lost bag but should expect to see more than one lost bag infrequently.

TABLE 6-6 Poisson Table for Various Values of μ (from Appendix B.5)

x	μ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

These probabilities can also be found using the Minitab system. The commands necessary are reported at the end of the chapter.

Worksheet1 ***						
	C1	C2	C3	C4	C5	C6
	Success	Probability				
1	0	0.740818				
2	1	0.222245				
3	2	0.033337				
4	3	0.003334				
5	4	0.000260				
6	5	0.000015				
7						





Earlier in this section, we mentioned that the Poisson probability distribution is a limiting form of the binomial. That is, we could estimate a binomial probability using the Poisson.

The Poisson probability distribution is characterized by the number of times an event happens during some interval or continuum. Examples include:

- The number of misspelled words per page in a newspaper.
- The number of calls per hour received by Dyson Vacuum Cleaner Company.
- The number of vehicles sold per day at Hyatt Buick GMC in Durham, North Carolina.
- The number of goals scored in a college soccer game.

In each of these examples, there is some type of continuum—misspelled words per page, calls per hour, vehicles per day, or goals per game.

In the previous example, we investigated the number of bags lost per flight, so the continuum was a “flight.” We knew the mean number of bags of luggage lost per flight, but we did not know the number of passengers or the probability of a bag being lost. We suspected the number of passengers was fairly large and the probability of a passenger losing his or her bag of luggage was small. In the following example, we use the Poisson distribution to estimate a binomial probability when n , the number of trials, is large and π , the probability of a success, small.

Example

Coastal Insurance Company underwrites insurance for beachfront properties along the Virginia, North and South Carolina, and Georgia coasts. It uses the estimate that the probability of a named Category III hurricane (sustained winds of more than 110 miles per hour) or higher striking a particular region of the coast (for example, St. Simons Island, Georgia) in any one year is .05. If a homeowner takes a 30-year mortgage on a recently purchased property in St. Simons, what is the likelihood that the owner will experience at least one hurricane during the mortgage period?

Solution

To use the Poisson probability distribution, we begin by determining the mean or expected number of storms meeting the criterion hitting St. Simons during the 30-year period. That is:

$$\mu = n\pi = 30(.05) = 1.5$$

where:

n is the number of years, 30 in this case.

π is the probability a hurricane meeting the strength criteria comes ashore.

μ is the mean or expected number of storms in a 30-year period.

To find the probability of at least one storm hitting St. Simons Island, Georgia, we first find the probability of no storms hitting the coast and subtract that value from 1.

$$P(x \geq 1) = 1 - P(x = 0) = 1 - \frac{\mu^0 e^{-\mu}}{0!} = 1 - .2231 = .7769$$

We conclude that the likelihood a hurricane meeting the strength criteria will strike the beachfront property at St. Simons during the 30-year period when the mortgage is in effect is .7769. To put it another way, the probability St. Simons will be hit by a Category III or higher hurricane during the 30-year period is a little more than 75 percent.

We should emphasize that the continuum, as previously described, still exists. That is, there are expected to be 1.5 storms hitting the coast per 30-year period. The continuum is the 30-year period.





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In the preceding case, we are actually using the Poisson distribution as an estimate of the binomial. Note that we've met the binomial conditions outlined on page 196.

- There are only two possible outcomes: a hurricane hits the St. Simons area or it does not.
- There is a fixed number of trials, in this case 30 years.
- There is a constant probability of success; that is, the probability of a hurricane hitting the area is .05 each year.
- The years are independent. That means if a named storm strikes in the fifth year, that has no effect on any other year.

To find the probability of at least one storm striking the area in a 30-year period using the binomial distribution:

$$P(x \geq 1) = 1 - P(x = 0) = 1 - {}_{30}C_0(.05)^0(.95)^{30} = 1 - (1)(.2146) = .7854$$

The probability of at least one hurricane hitting the St. Simons area during the 30-year period using the binomial distribution is .7854.

Which answer is correct? Why should we look at the problem both ways? The binomial is the more "technically correct" solution. The Poisson can be thought of as an approximation for the binomial, when n , the number of trials is large, and π , the probability of a success, is small. We look at the problem using both distributions to emphasize the convergence of the two discrete distributions. In some instances, using the Poisson may be the quicker solution, and as you see there is little practical difference in the answers. In fact, as n gets larger and π smaller, the differences between the two distributions gets smaller.

The Poisson probability distribution is always positively skewed and the random variable has no specific upper limit. The Poisson distribution for the lost bags illustration, where $\mu = 0.3$, is highly skewed. As μ becomes larger, the Poisson distribution becomes more symmetrical. For example, Chart 6-4 shows the distributions of the number of transmission services, muffler replacements, and oil changes per day at Avellino's Auto Shop. They follow Poisson distributions with means of 0.7, 2.0, and 6.0, respectively.

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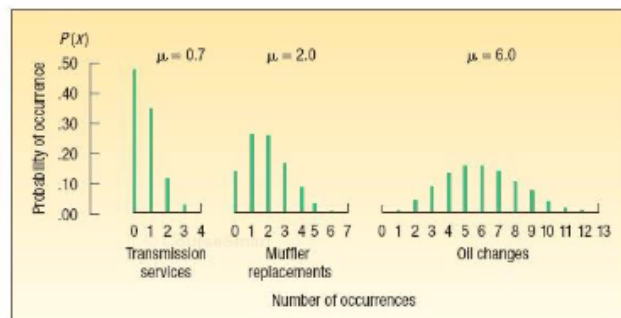


CHART 6-4 Poisson Probability Distributions for Means of 0.7, 2.0, and 6.0

Only μ needed to construct Poisson distribution

In summary, the Poisson distribution is actually a family of discrete distributions. All that is needed to construct a Poisson probability distribution is the mean number of defects, errors, and so on—designated as μ .



**Self-Review 6-6.77**

From actuary tables, Washington Insurance Company determined the likelihood that a man age 25 will die within the next year is .0002. If Washington Insurance sells 4,000 policies to 25-year-old men this year, what is the probability they will pay on exactly one policy?

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Exercises

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31. In a Poisson distribution $\mu = 0.4$.
 - a. What is the probability that $x = 0$?
 - b. What is the probability that $x > 0$?
32. In a Poisson distribution $\mu = 4$.
 - a. What is the probability that $x = 2$?
 - b. What is the probability that $x \leq 2$?
 - c. What is the probability that $x > 2$?
33. Ms. Bergen is a loan officer at Coast Bank and Trust. From her years of experience, she estimates that the probability is .025 that an applicant will not be able to repay his or her installment loan. Last month she made 40 loans.
 - a. What is the probability that 3 loans will be defaulted?
 - b. What is the probability that at least 3 loans will be defaulted?
34. Automobiles arrive at the Elkhart exit of the Indiana Toll Road at the rate of two per minute. The distribution of arrivals approximates a Poisson distribution.
 - a. What is the probability that no automobiles arrive in a particular minute?
 - b. What is the probability that at least one automobile arrives during a particular minute?
35. It is estimated that 0.5 percent of the callers to the Customer Service department of Dell Inc. will receive a busy signal. What is the probability that of today's 1,200 callers at least 5 received a busy signal?
36. In the past, schools in Los Angeles County have closed an average of three days each year for weather emergencies. What is the probability that schools in Los Angeles County will close for four days next year?

Chapter Summary

- I. A random variable is a numerical value determined by the outcome of an experiment.
- II. A probability distribution is a listing of all possible outcomes of an experiment and the probability associated with each outcome.
 - A. A discrete probability distribution can assume only certain values. The main features are:
 1. The sum of the probabilities is 1.00.
 2. The probability of a particular outcome is between 0.00 and 1.00.
 3. The outcomes are mutually exclusive.
 - B. A continuous distribution can assume an infinite number of values within a specific range.
- III. The mean and variance of a probability distribution are computed as follows.
 - A. The mean is equal to:

$$\mu = \sum [xP(x)] \quad [6-1]$$
 - B. The variance is equal to:

$$\sigma^2 = \sum [(x - \mu)^2 P(x)] \quad [6-2]$$
- IV. The binomial distribution has the following characteristics.
 - A. Each outcome is classified into one of two mutually exclusive categories.
 - B. The distribution results from a count of the number of successes in a fixed number of trials.

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Discrete Probability Distributions

213

CourseSmart #7347718

- C. The probability of a success remains the same from trial to trial.
- D. Each trial is independent.
- E. A binomial probability is determined as follows:

$$P(x) = {}_n C_x \pi^x (1 - \pi)^{n-x} \quad [6-3]$$

- F. The mean is computed as:

$$\mu = n\pi \quad [6-4]$$

- G. The variance is

$$\sigma^2 = n\pi(1 - \pi) \quad [6-5]$$

- V. The hypergeometric distribution has the following characteristics.

- A. There are only two possible outcomes.
- B. The probability of a success is not the same on each trial.
- C. The distribution results from a count of the number of successes in a fixed number of trials.
- D. It is used when sampling without replacement from a finite population.
- E. A hypergeometric probability is computed from the following equation:

$$P(x) = \frac{{}_r C_x {}_{N-r} C_{n-x}}{{}_N C_n} \quad [6-6]$$

- VI. The Poisson distribution has the following characteristics.

- A. It describes the number of times some event occurs during a specified interval.
- B. The probability of a "success" is proportional to the length of the interval.
- C. Nonoverlapping intervals are independent.
- D. It is a limiting form of the binomial distribution when n is large and π is small.
- E. A Poisson probability is determined from the following equation:

$$P(x) = \frac{\mu^x e^{-\mu}}{x!} \quad [6-7]$$



- F. The mean and the variance are:

$$\mu = n\pi \quad [6-8]$$

$$\sigma^2 = n\pi$$

Chapter Exercises

connect


37. What is the difference between a random variable and a probability distribution?
38. For each of the following indicate whether the random variable is discrete or continuous.
 - a. The length of time to get a haircut.
 - b. The number of cars a jogger passes each morning while running.
 - c. The number of hits for a team in a high school girls' softball game.
 - d. The number of patients treated at the South Strand Medical Center between 6 and 10 p.m. each night.
 - e. The distance your car traveled on the last fill-up.
 - f. The number of customers at the Oak Street Wendy's who used the drive-through facility.
 - g. The distance between Gainesville, Florida, and all Florida cities with a population of at least 50,000.
39. An investment will be worth \$1,000, \$2,000, or \$5,000 at the end of the year. The probabilities of these values are .25, .60, and .15, respectively. Determine the mean and variance of the worth of the investment. 
40. The personnel manager of Cumberland Pig Iron Company is studying the number of on-the-job accidents over a period of one month. He developed the following probability distribution. Compute the mean, variance, and standard deviation of the number of accidents in a month. 

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Number of Accidents	Probability
0	.40
1	.20
2	.20
3	.10
4	.10


41. Croissant Bakery Inc. offers special decorated cakes for birthdays, weddings, and other occasions. It also has regular cakes available in its bakery. The following table gives the total number of cakes sold per day and the corresponding probability. Compute the mean, variance, and standard deviation of the number of cakes sold per day. 

Number of Cakes Sold in a Day	Probability
12	.25
13	.40
14	.25
15	.10

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42. The payouts for the Powerball lottery and their corresponding odds and probabilities of occurrence are shown below. The price of a ticket is \$1.00. Find the mean and standard deviation of the payout. Hint: Don't forget to include the cost of the ticket and its corresponding probability. 

Divisions	Payout	Odds	Probability
Five plus Powerball	\$50,000,000	146,107,962	0.00000006844
Match 5	200,000	3,563,609	0.00000280614
Four plus Powerball	10,000	584,432	0.00001711069
Match 4	100	14,255	0.000070145903
Three plus Powerball	100	11,927	0.000083836351
Match 3	7	291	0.003424657534
Two plus Powerball	7	745	0.001340482574
One plus Powerball	4	127	0.007812500000
Zero plus Powerball	3	69	0.014285714286

43. In a recent survey, 35 percent indicated chocolate was their favorite flavor of ice cream. Suppose we select a sample of ten people and ask them to name their favorite flavor of ice cream.
- How many of those in the sample would you expect to name chocolate?
 - What is the probability exactly four of those in the sample name chocolate?
 - What is the probability four or more name chocolate?
44. Thirty percent of the population in a southwestern community are Spanish-speaking Americans. A Spanish-speaking person is accused of killing a non-Spanish-speaking American and goes to trial. Of the first 12 potential jurors, only 2 are Spanish-speaking Americans, and 10 are not. The defendant's lawyer challenges the jury selection, claiming bias against her client. The government lawyer disagrees, saying that the probability of this particular jury composition is common. Compute the probability and discuss the assumptions. 
45. An auditor for Health Maintenance Services of Georgia reports 40 percent of policyholders 55 years or older submit a claim during the year. Fifteen policyholders are randomly selected for company records.
- How many of the policyholders would you expect to have filed a claim within the last year?
 - What is the probability that 10 of the selected policyholders submitted a claim last year?

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Discrete Probability Distributions

215

CourseSmart #7347718

- c. What is the probability that 10 or more of the selected policyholders submitted a claim last year?
- d. What is the probability that more than 10 of the selected policyholders submitted a claim last year?
- 46. Tire and Auto Supply is considering a 2-for-1 stock split. Before the transaction is finalized, at least two-thirds of the 1,200 company stockholders must approve the proposal. To evaluate the likelihood the proposal will be approved, the CFO selected a sample of 18 stockholders. He contacted each and found 14 approved of the proposed split. What is the likelihood of this event, assuming two-thirds of the stockholders approve?
- 47. A federal study reported that 7.5 percent of the U.S. workforce has a drug problem. A drug enforcement official for the State of Indiana wished to investigate this statement. In her sample of 20 employed workers:
 - a. How many would you expect to have a drug problem? What is the standard deviation?
 - b. What is the likelihood that *none* of the workers sampled has a drug problem?
 - c. What is the likelihood that *at least one* has a drug problem?
- 48. The Bank of Hawaii reports that 7 percent of its credit card holders will default at some time in their life. The Hilo branch just mailed out 12 new cards today.
 - a. How many of these new cardholders would you expect to default? What is the standard deviation?
 - b. What is the likelihood that *none* of the cardholders will default?
 - c. What is the likelihood that *at least one* will default?
- 49. Recent statistics suggest that 15 percent of those who visit a retail site on the World Wide Web make a purchase. A retailer wished to verify this claim. To do so, she selected a sample of 16 "hits" to her site and found that 4 had actually made a purchase.
 - a. What is the likelihood of exactly four purchases?
 - b. How many purchases should she expect?
 - c. What is the likelihood that four or more "hits" result in a purchase?
- 50. In Chapter 19, we discuss the acceptance sample. Acceptance sampling is used to monitor the quality of incoming raw materials. Suppose a purchaser of electronic components allows 1 percent of the components to be defective. To ensure the quality of incoming parts, a purchaser or manufacturer normally samples 20 parts and allows 1 defect.
 - a. What is the likelihood of accepting a lot that is 1 percent defective?
 - b. If the quality of the incoming lot was actually 2 percent, what is the likelihood of accepting it?
 - c. If the quality of the incoming lot was actually 5 percent, what is the likelihood of accepting it?
- 51. Colgate-Palmolive Inc. recently developed a new toothpaste flavored with honey. It tested a group of ten people. Six of the group said they liked the new flavor, and the remaining four indicated they definitely did not. Four of the ten are selected to participate in an in-depth interview. What is the probability that of those selected for the in-depth interview two liked the new flavor and two did not?
- 52. Dr. Richmond, a psychologist, is studying the daytime television viewing habits of college students. She believes 45 percent of college students watch soap operas during the afternoon. To further investigate, she selects a sample of 10.
 - a. Develop a probability distribution for the number of students in the sample who watch soap operas.
 - b. Find the mean and the standard deviation of this distribution.
 - c. What is the probability of finding exactly four watch soap operas?
 - d. What is the probability less than half of the students selected watch soap operas?
- 53. A recent study conducted by Penn, Shone, and Borland, on behalf of LastMinute.com, revealed that 52 percent of business travelers plan their trips less than two weeks before departure. The study is to be replicated in the tri-state area with a sample of 12 frequent business travelers.
 - a. Develop a probability distribution for the number of travelers who plan their trips within two weeks of departure.
 - b. Find the mean and the standard deviation of this distribution.
 - c. What is the probability exactly 5 of the 12 selected business travelers plan their trips within two weeks of departure?
 - d. What is the probability 5 or fewer of the 12 selected business travelers plan their trips within two weeks of departure?

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54. Suppose the Internal Revenue Service is studying the category of charitable contributions. A sample of 25 returns is selected from young couples between the ages of 20 and 35 who had an adjusted gross income of more than \$100,000. Of these 25 returns, five had charitable contributions of more than \$1,000. Suppose four of these returns are selected for a comprehensive audit.
- Explain why the hypergeometric distribution is appropriate.
 - What is the probability exactly one of the four audited had a charitable deduction of more than \$1,000?
 - What is the probability at least one of the audited returns had a charitable contribution of more than \$1,000?
55. The law firm of Hagel and Hagel is located in downtown Cincinnati. There are 10 partners in the firm; 7 live in Ohio and 3 in northern Kentucky. Ms. Wendy Hagel, the managing partner, wants to appoint a committee of 3 partners to look into moving the firm to northern Kentucky. If the committee is selected at random from the 10 partners, what is the probability that:
- One member of the committee lives in northern Kentucky and the others live in Ohio?
 - At least 1 member of the committee lives in northern Kentucky?
56. Recent information published by the U.S. Environmental Protection Agency indicates that Honda is the manufacturer of four of the top nine vehicles in terms of fuel economy.
- Determine the probability distribution for the number of Hondas in a sample of three cars chosen from the top nine.
 - What is the likelihood that in the sample of three at least one Honda is included?
57. The position of chief of police in the city of Corry, Pennsylvania, is vacant. A search committee of Corry residents is charged with the responsibility of recommending a new chief to the city council. There are 12 applicants, 4 of which are either female or members of a minority. The search committee decides to interview all 12 of the applicants. To begin, they randomly select four applicants to be interviewed on the first day, and none of the four is female or a member of a minority. The local newspaper, the Corry Press, suggests discrimination in an editorial. What is the likelihood of this occurrence?
58. Listed below is the population by state for the 15 states with the largest population. Also included is whether that state's border touches the Gulf of Mexico, the Atlantic Ocean, or the Pacific Ocean (coastline).

Rank	State	Population	Coastline
1	California	36,553,215	Yes
2	Texas	23,904,380	Yes
3	New York	19,297,729	Yes
4	Florida	18,251,243	Yes
5	Illinois	12,852,548	No
6	Pennsylvania	12,432,792	No
7	Ohio	11,466,917	No
8	Michigan	10,071,822	No
9	Georgia	9,544,750	Yes
10	North Carolina	9,061,032	Yes
11	New Jersey	8,685,920	Yes
12	Virginia	7,712,091	Yes
13	Washington	6,468,424	Yes
14	Massachusetts	6,449,755	Yes
15	Indiana	6,345,289	No

Note that 5 of the 15 states do not have any coastline. Suppose three states are selected at random. What is the probability that:

- None of the states selected have any coastline?
- Exactly one of the selected states has a coastline?
- At least one of the selected states has a coastline?





Discrete Probability Distributions

217

CourseSmart #7347718

59. The sales of Lexus automobiles in the Detroit area follow a Poisson distribution with a mean of 3 per day.
- What is the probability that no Lexus is sold on a particular day?
 - What is the probability that for five consecutive days at least one Lexus is sold?
60. Suppose 1.5 percent of the antennas on new Nokia cell phones are defective. For a random sample of 200 antennas, find the probability that:
- None of the antennas is defective.
 - Three or more of the antennas are defective.
61. A study of the checkout lines at the Safeway Supermarket in the South Strand area revealed that between 4 and 7 P.M. on weekdays there is an average of four customers waiting in line. What is the probability that you visit Safeway today during this period and find:
- No customers are waiting?
 - Four customers are waiting?
 - Four or fewer are waiting?
 - Four or more are waiting?
62. An internal study by the Technology Services department at Lahey Electronics revealed company employees receive an average of two emails per hour. Assume the arrival of these emails is approximated by the Poisson distribution.
- What is the probability Linda Lahey, company president, received exactly 1 email between 4 P.M. and 5 P.M. yesterday?
 - What is the probability she received 5 or more emails during the same period?
 - What is the probability she did not receive any emails during the period?
63. Recent crime reports indicate that 3.1 motor vehicle thefts occur each minute in the United States. Assume that the distribution of thefts per minute can be approximated by the Poisson probability distribution.
- Calculate the probability exactly four thefts occur in a minute.
 - What is the probability there are no thefts in a minute?
 - What is the probability there is at least one theft in a minute?
64. New Process Inc., a large mail-order supplier of women's fashions, advertises same-day service on every order. Recently, the movement of orders has not gone as planned, and there were a large number of complaints. Bud Owens, director of customer service, has completely redone the method of order handling. The goal is to have fewer than five unfilled orders on hand at the end of 95 percent of the working days. Frequent checks of the unfilled orders at the end of the day reveal that the distribution of the unfilled orders follows a Poisson distribution with a mean of two orders.
- Has New Process Inc. lived up to its internal goal? Cite evidence.
 - Draw a histogram representing the Poisson probability distribution of unfilled orders.
65. The National Aeronautics and Space Administration (NASA) has experienced two disasters. The Challenger exploded over the Atlantic Ocean in 1986, and the Columbia disintegrated on reentry over East Texas in 2003. Based on the first 113 missions, and assuming failures occur at the same rate, consider the next 23 missions. What is the probability of exactly two failures? What is the probability of no failures?
66. According to the "January theory," if the stock market is up for the month of January, it will be up for the year. If it is down in January, it will be down for the year. According to an article in *The Wall Street Journal*, this theory held for 29 out of the last 34 years. Suppose there is no truth to this theory; that is, the probability it is either up or down is .50. What is the probability this could occur by chance? You will probably need a software package such as Excel or Minitab.
67. During the second round of the 1989 U.S. Open golf tournament, four golfers scored a hole in one on the sixth hole. The odds of a professional golfer making a hole in one are estimated to be 3,708 to 1, so the probability is $1/3,709$. There were 155 golfers participating in the second round that day. Estimate the probability that four golfers would score a hole in one on the sixth hole.
68. Suppose the National Hurricane Center forecasts that hurricanes will hit the strike area with a .95 probability. Answer the following questions:
- What probability distribution does this follow?
 - What is the probability that 10 hurricanes reach landfall in the strike area?
 - What is the probability at least one of 10 hurricanes reaches land outside the strike area?



**STORM CONTINUES
NORTHWEST**

Position : 27.8 N, 71.4 W
Movement: NNW at 8 mph
Sustained winds: 105 mph
As of 11 p.m. EDT Tuesday

— Hurricane watch
— Tropical storm watch



69. A recent CBS News survey reported that 67 percent of adults felt the U.S. Treasury should continue making pennies.



Suppose we select a sample of 15 adults.

- How many of the 15 would we expect to indicate that the Treasury should continue making pennies? What is the standard deviation?
- What is the likelihood that exactly 8 adults would indicate the Treasury should continue making pennies?
- What is the likelihood at least 8 adults would indicate the Treasury should continue making pennies?

Data Set Exercises

- Refer to the Real Estate data, which report information on homes sold in the Goodyear, Arizona, area last year.
 - Create a probability distribution for the number of bedrooms. Compute the mean and the standard deviation of this distribution.
 - Create a probability distribution for the number of bathrooms. Compute the mean and the standard deviation of this distribution.
- Refer to the Baseball 2009 data. Compute the mean number of home runs per game. To do this, first find the mean number of home runs per team for 2009. Next, divide this value by 162 (a season comprises 162 games). Then multiply by 2, because there are two teams in each game. Use the Poisson distribution to estimate the number of home runs that will be hit in a game. Find the probability that:
 - There are no home runs in a game.
 - There are two home runs in a game.
 - There are at least four home runs in a game.

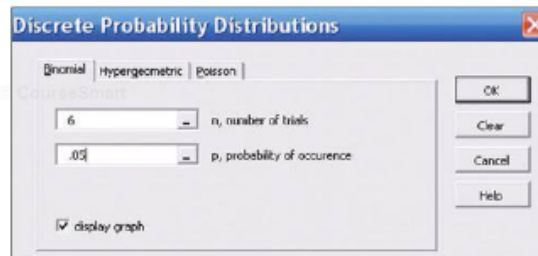




Software Commands

1. The MegaStat commands to create the binomial probability distribution on page 199 are:
 - a. Select the **Add-Ins** tab in the top menu. On the far left, select the **MegaStat** pull-down menu.

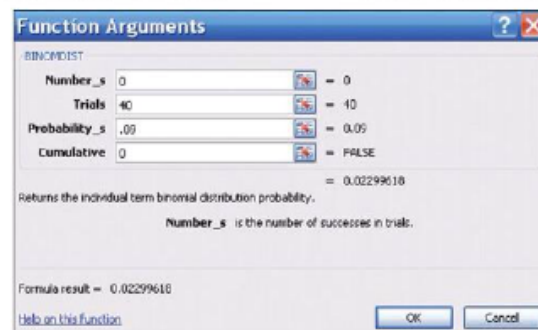
Click on **probability**, and **Discrete Probability Distributions**. Enter **n**, number of trials, and **p**, probability of occurrence, and click **OK**.



- b. In the dialog box select the **Binomial** tab. The number of trials is 6, the probability of a success is .05. If you wish to see a graph, click on **display graph**.
2. The Excel commands necessary to determine the binomial probability distribution on page 200 are:
 - a. On a blank Excel worksheet, write the word **Success** in cell A1 and the word **Probability** in B1. In cells A2 through A17, write the integers 0 to 15. Click on B2 as the active cell.
 - b. Click on the **Formulas** tab in the top menu, then, on the far left, select **Insert Function fx**.
 - c. In the first dialog box, select **Statistical** in the function category and **BINOMDIST** in the function name category, then click **OK**.
 - d. In the second dialog box, enter the four items necessary to compute a binomial probability.
 1. Enter 0 for the number of successes.
 2. Enter 40 for the number of trials.
 3. Enter .09 for the probability of a success.
 4. Enter the word **false** or the number 0 for the individual probabilities and click on **OK**.

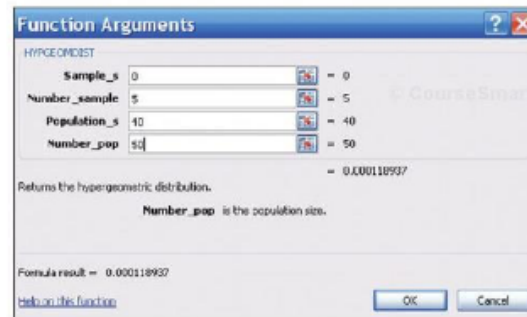
5. Excel will compute the probability of 0 successes in 40 trials, with a .09 probability of success. The result, .02299618, is stored in cell B2.

- e. To complete the probability distribution for successes of 1 through 15, double click on cell B2. The binomial function should appear. Replace the 0 to the right of the open parentheses with the cell reference A2.
- f. Move the mouse to the lower right corner of cell B2 till a solid black + symbol appears, then click and hold and highlight the B column to cell B17. The probability of a success for the various values of the random variable will appear.
3. The Excel commands necessary to determine the hypergeometric distribution on page 206 are:
 - a. On a blank Excel worksheet, write the word **Union Members** in cell A1 and the word **Probability** in B1. In cells A2 through A7, enter the numbers 0 through 5. Click on cell B2.
 - b. Click the **Formulas** tab in the top menu, then, on the far left, select **Insert Function fx**.

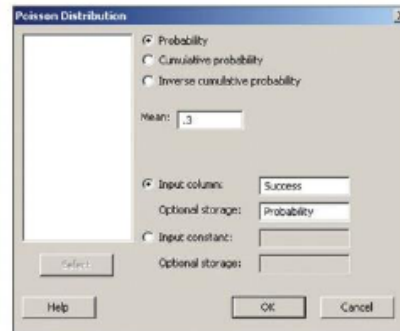




- c. In the first dialog box, select **Statistical** and **HYPGEOMDIST**, and then click **OK**.
- d. In the second dialog box, enter the four items necessary to compute a hypergeometric probability.
 1. Enter 0 for the number of successes.
 2. Enter 5 for the number of trials.
 3. Enter 40 for the number of successes in the population.
 4. Enter 50 for the size of the population and click **OK**.
- e. To complete the probability distribution for successes of 1 through 5, double click on cell **B2**. The hypergeometric function should appear. Replace the 0 to the right of the open parentheses with the cell reference **A2**.
- f. Move the mouse to the lower right corner of cell **F9** till a solid black + symbol appears, then click and hold and highlight the **F** column to cell **F14**. The probability of a success for the various outcomes will appear.



4. The Minitab commands to generate the Poisson distribution on page 209 are:
 - a. Label column **C1** as **Successes** and **C2** as **Probability**. Enter the integers 0 through 5 in the first column.
 - b. Select **Calc**, then **Probability Distributions**, and **Poisson**.
 - c. In the dialog box, click on **Probability**, set the mean equal to .3, and select **C1** as the Input column. Designate **C2** as Optional storage, and then click **OK**.



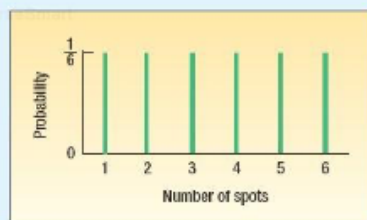


Chapter 6 Answers to Self-Review

6-1 a.

Number of Spots	Probability
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$
Total	$\frac{6}{6} = 1.00$

b.

c. $\frac{6}{6}$ or 1.

- 6-2 a. It is discrete, because the value \$0.80, \$0.90, and \$1.20 are clearly separated from each other. Also the sum of the probabilities is 1.00, and the outcomes are mutually exclusive.

b.

x	$P(x)$	$xP(x)$
\$.80	.30	0.24
.90	.50	0.45
1.20	.20	0.24
		0.93

The mean is 93 cents.

c.

x	$P(x)$	$(x - \mu)$	$(x - \mu)^2 P(x)$
\$0.80	.30	-0.13	.00507
0.90	.50	-0.03	.00045
1.20	.20	0.27	.01458
			.02010

The variance is .02010, and the standard deviation is 14 cents.

6-3

- a. It is reasonable because each employee either uses direct deposit or does not; employees are independent; the probability of using direct deposit is .80 for all; and we count the number using the service out of 7.

$$b. P(7) = {}_7C_7 (.80)^7 (.20)^0 = .2097$$

$$c. P(4) = {}_7C_4 (.80)^4 (.20)^3 = .1147$$

d. Answers are in agreement.

6-4

$$n = 4, \pi = .60$$

$$a. P(x = 2) = .346$$

$$b. P(x \leq 2) = .526$$

$$c. P(x > 2) = 1 - .526 = .474$$

6-5

$$P(3) = \frac{{}_9C_3 {}_4C_2}{12C_5} = \frac{\left(\frac{8!}{3!5!}\right)\left(\frac{4!}{2!2!}\right)}{\frac{12!}{5!7!}}$$

$$= \frac{(56)(6)}{792} = .424$$

6-6

$$\mu = 4,000(.0002) = 0.8$$

$$P(1) = \frac{0.8^1 e^{-0.8}}{1!} = .3595$$





7

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Learning Objectives

When you have completed this chapter, you will be able to:

L01 List the characteristics of the uniform distribution.

L02 Compute probabilities using the uniform distribution.

L03 List the characteristics of the normal distribution.

L04 Convert a normal distribution to the standard normal distribution.

L05 Find the probability that a normally distributed random variable is between two values.

L06 Find probabilities using the Empirical Rule.

L07 Approximate the binomial distribution using the normal distribution.

L08 Describe the characteristics and compute probabilities using the exponential distribution.

Continuous Probability Distributions

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Cruise ships of the Royal Viking line report that 80 percent of their rooms are occupied during September. For a cruise ship having 800 rooms, what is the probability that 665 or more are occupied in September? (See Exercise 60 and L07.)

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7.1 Introduction

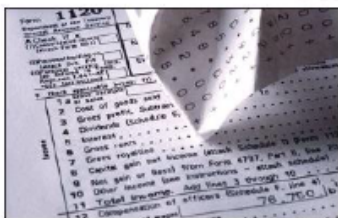
Chapter 6 began our study of probability distributions. We consider three *discrete* probability distributions: binomial, hypergeometric, and Poisson. These distributions are based on discrete random variables, which can assume only clearly separated values. For example, we select for study 10 small businesses that began operations during the year 2000. The number still operating in 2011 can be 0, 1, 2, ..., 10. There cannot be 3.7, 12, or -7 still operating in 2011. In this example, only certain outcomes are possible and these outcomes are represented by clearly separated values. In addition, the result is usually found by counting the number of successes. We count the number of the businesses in the study that are still in operation in 2011.

In this chapter, we continue our study of probability distributions by examining *continuous* probability distributions. A continuous probability distribution usually results from measuring something, such as the distance from the dormitory to the classroom, the weight of an individual, or the amount of bonus earned by CEOs. Suppose we select five students and find the distance, in miles, they travel to attend class as 12.2, 8.9, 6.7, 3.6, and 14.6. When examining a continuous distribution we are usually interested in information such as the percent of students who travel less than 10 miles or the percent who travel more than 8 miles. In other words, for a continuous distribution we may wish to know the percent of observations that occur within a certain range. It is important to realize that a continuous random variable has an infinite number of values within a particular range. So you think of the probability a variable will have a value within a specified range, rather than the probability for a specific value.

We consider three families of continuous probability distributions, the **uniform probability distribution**, the normal probability distribution, and the exponential probability distribution.

7.2 The Family of Uniform Probability Distributions

The uniform probability distribution is perhaps the simplest distribution for a continuous random variable. This distribution is rectangular in shape and is defined by minimum and maximum values. Here are some examples that follow a uniform distribution.



- The time to fly via a commercial airliner from Orlando, Florida, to Atlanta, Georgia, ranges from 60 minutes to 120 minutes. The random variable is the flight time within this interval. The variable of interest, flight time in minutes, is continuous in the interval from 60 minutes to 120 minutes.
- Volunteers at the Grand Strand Public Library prepare federal income tax forms. The time to prepare form 1040-EZ follows a uniform distribution over the interval between 10 minutes and 30 minutes. The random variable is the number of minutes to complete the form, and it can assume any value between 10 and 30.

A uniform distribution is shown in Chart 7-1. The distribution's shape is rectangular and has a minimum value of a and a maximum of b . Also notice in Chart 7-1 the height of the distribution is constant or uniform for all values between a and b .





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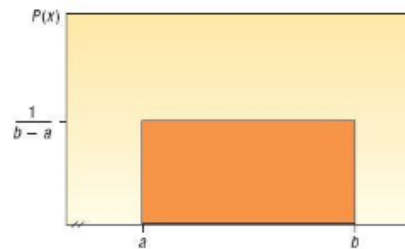


CHART 7-1 A Continuous Uniform Distribution

The mean of a uniform distribution is located in the middle of the interval between the minimum and maximum values. It is computed as:

MEAN OF THE UNIFORM DISTRIBUTION

$$\mu = \frac{a + b}{2}$$

[7-1]

LO1 List the characteristics of the uniform distribution.

The standard deviation describes the dispersion of a distribution. In the uniform distribution, the standard deviation is also related to the interval between the maximum and minimum values.

STANDARD DEVIATION OF THE UNIFORM DISTRIBUTION

$$\sigma = \sqrt{\frac{(b - a)^2}{12}}$$

[7-2]

The equation for the uniform probability distribution is:

UNIFORM DISTRIBUTION

$$P(x) = \frac{1}{b - a} \quad \text{if } a \leq x \leq b \text{ and } 0 \text{ elsewhere}$$

[7-3]

The total area under the curve is always 1.

As described in Chapter 6, probability distributions are useful for making probability statements concerning the values of a random variable. For distributions describing a continuous random variable, areas within the distribution represent probabilities. In the uniform distribution, its rectangular shape allows us to apply the area formula for a rectangle. Recall that we find the area of a rectangle by multiplying its length by its height. For the uniform distribution, the height of the rectangle is $P(x)$, which is $1/(b - a)$. The length or base of the distribution is $b - a$. So if we multiply the height of the distribution by its entire range to find the area, the result is always 1.00. To put it another way, the total area within a continuous probability distribution is equal to 1.00. In general

$$\text{Area} = (\text{height})(\text{base}) = \frac{1}{(b - a)} (b - a) = 1.00$$

So if a uniform distribution ranges from 10 to 15, the height is 0.20, found by $1/(15 - 10)$. The base is 5, found by $15 - 10$. The total area is:

$$\text{Area} = (\text{height})(\text{base}) = \frac{1}{(15 - 10)} (15 - 10) = 1.00$$

An example will illustrate the features of a uniform distribution and how we calculate probabilities using it.



**Example**

Southwest Arizona State University provides bus service to students while they are on campus. A bus arrives at the North Main Street and College Drive stop every 30 minutes between 6 A.M. and 11 P.M. during weekdays. Students arrive at the bus stop at random times. The time that a student waits is uniformly distributed from 0 to 30 minutes.

1. Draw a graph of this distribution.
2. Show that the area of this uniform distribution is 1.00.
3. How long will a student "typically" have to wait for a bus? In other words, what is the mean waiting time? What is the standard deviation of the waiting times?
4. What is the probability a student will wait more than 25 minutes?
5. What is the probability a student will wait between 10 and 20 minutes?

Solution

In this case, the random variable is the length of time a student must wait. Time is measured on a continuous scale, and the wait times may range from 0 minutes up to 30 minutes.

1. The graph of the uniform distribution is shown in Chart 7-2. The horizontal line is drawn at a height of .0333, found by $1/(30 - 0)$. The range of this distribution is 30 minutes.

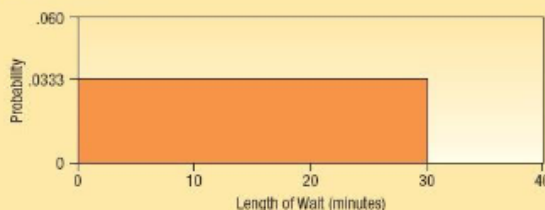


CHART 7-2 Uniform Probability Distribution of Student Waiting Times

2. The times students must wait for the bus is uniform over the interval from 0 minutes to 30 minutes, so in this case a is 0 and b is 30.

$$\text{Area} = (\text{height})(\text{base}) = \frac{1}{(30 - 0)}(30 - 0) = 1.00$$

3. To find the mean, we use formula (7-1).

$$\mu = \frac{a + b}{2} = \frac{0 + 30}{2} = 15$$

The mean of the distribution is 15 minutes, so the typical wait time for bus service is 15 minutes.

To find the standard deviation of the wait times, we use formula (7-2).

$$\sigma = \sqrt{\frac{(b - a)^2}{12}} = \sqrt{\frac{(30 - 0)^2}{12}} = 8.66$$

The standard deviation of the distribution is 8.66 minutes. This measures the variation in the student wait times.

4. The area within the distribution for the interval 25 to 30 represents this particular probability. From the area formula:

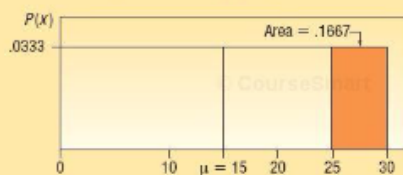
$$P(25 < \text{wait time} < 30) = (\text{height})(\text{base}) = \frac{1}{(30 - 0)}(5) = .1667$$

L02 Compute probabilities using the uniform distribution.





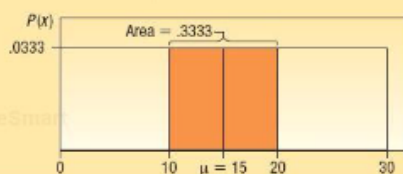
CourseSmart #7 So the probability a student waits between 25 and 30 minutes is .1667. This conclusion is illustrated by the following graph.



5. The area within the distribution for the interval 10 to 20 represents the probability.

$$P(10 < \text{wait time} < 20) = (\text{height})(\text{base}) = \frac{1}{(30 - 0)}(10) = .3333$$

We can illustrate this probability as follows.



Self-Review 7-1



Australian sheepdogs have a relatively short life. The length of their life follows a uniform distribution between 6 and 14 years.

- Draw this uniform distribution. What are the height and base values?
- Show the total area under the curve is 1.00.
- Calculate the mean and the standard deviation of this distribution.
- What is the probability a particular dog lives between 10 and 14 years?
- What is the probability a dog will live less than 9 years?

Exercises

connect

- A uniform distribution is defined over the interval from 6 to 10.
 - What are the values for a and b ?
 - What is the mean of this uniform distribution?
 - What is the standard deviation?
 - Show that the total area is 1.00.
 - Find the probability of a value more than 7.
 - Find the probability of a value between 7 and 9.
- A uniform distribution is defined over the interval from 2 to 5.
 - What are the values for a and b ?
 - What is the mean of this uniform distribution?
 - What is the standard deviation?
 - Show that the total area is 1.00.
 - Find the probability of a value more than 2.6.
 - Find the probability of a value between 2.9 and 3.7.
- The closing price of Schnur Sporting Goods Inc. common stock is uniformly distributed between \$20 and 30 per share. What is the probability that the stock price will be:
 - More than \$27?
 - Less than or equal to \$24?





- CourseSmart #7
4. According to the Insurance Institute of America, a family of four spends between \$400 and \$3,600 per year on all types of insurance. Suppose the money spent is uniformly distributed between these amounts.
 - a. What is the mean amount spent on insurance?
 - b. What is the standard deviation of the amount spent?
 - c. If we select a family at random, what is the probability they spend less than \$2,000 per year on insurance per year?
 - d. What is the probability a family spends more than \$3,000 per year?
 5. The April rainfall in Flagstaff, Arizona, follows a uniform distribution between 0.5 and 3.00 inches.
 - a. What are the values for a and b ?
 - b. What is the mean amount of rainfall for the month? What is the standard deviation?
 - c. What is the probability of less than an inch of rain for the month?
 - d. What is the probability of exactly 1.00 inch of rain?
 - e. What is the probability of more than 1.50 inches of rain for the month?
 6. Customers experiencing technical difficulty with their Internet cable hookup may call an 800 number for technical support. It takes the technician between 30 seconds to 10 minutes to resolve the problem. The distribution of this support time follows the uniform distribution.
 - a. What are the values for a and b in minutes?
 - b. What is the mean time to resolve the problem? What is the standard deviation of the time?
 - c. What percent of the problems take more than 5 minutes to resolve?
 - d. Suppose we wish to find the middle 50 percent of the problem-solving times. What are the end points of these two times?

7.3 The Family of Normal Probability Distributions

Next we consider the normal probability distribution. Unlike the uniform distribution [see formula (7-3)] the normal probability distribution has a very complex formula.

NORMAL PROBABILITY DISTRIBUTION

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]} \quad [7-4]$$

However, do not be bothered by how complex this formula looks. You are already familiar with many of the values. The symbols μ and σ refer to the mean and the standard deviation, as usual. The Greek symbol π is a natural mathematical constant and its value is approximately 22/7 or 3.1416. The letter e is also a mathematical constant. It is the base of the natural log system and is equal to 2.718. X is the value of a continuous random variable. So a normal distribution is based on—that is, it is defined by—its mean and standard deviation.

You will not need to make any calculations from formula (7-4). Instead you will be using a table, which is given as Appendix B.1, to look up the various probabilities.

The normal probability distribution has the following major characteristics:

LO3 List the characteristics of the normal distribution.

- It is **bell-shaped** and has a single peak at the center of the distribution. The arithmetic mean, median, and mode are equal and located in the center of the distribution. The total area under the curve is 1.00. Half the area under the normal curve is to the right of this center point and the other half to the left of it.
- It is **symmetrical** about the mean. If we cut the normal curve vertically at the center value, the two halves will be mirror images.
- It falls off **smoothly** in either direction from the central value. That is, the distribution is **asymptotic**: The curve gets closer and closer to the X -axis but never actually touches it. To put it another way, the tails of the curve extend indefinitely in both directions.
- The location of a normal distribution is determined by the mean, μ . The dispersion or spread of the distribution is determined by the standard deviation, σ .

These characteristics are shown graphically in Chart 7-3.





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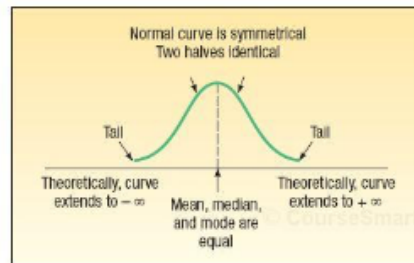


CHART 7-3 Characteristics of a Normal Distribution

There is not just one normal probability distribution, but rather a "family" of them. For example, in Chart 7-4 the probability distributions of length of employee service in three different plants can be compared. In the Camden plant, the mean is 20 years and the standard deviation is 3.1 years. There is another normal probability distribution for the length of service in the Dunkirk plant, where $\mu = 20$ years and $\sigma = 3.9$ years. In the Elmira plant, $\mu = 20$ years and $\sigma = 5.0$ years. Note that the means are the same but the standard deviations are different.

Equal means, unequal standard deviations

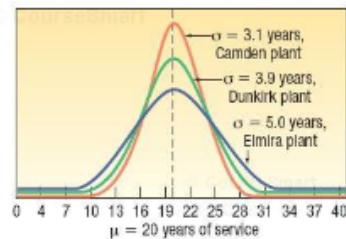


CHART 7-4 Normal Probability Distributions with Equal Means but Different Standard Deviations

Chart 7-5 shows the distribution of box weights of three different cereals. The weights follow a normal distribution with different means but identical standard deviations.

Unequal means, equal standard deviations

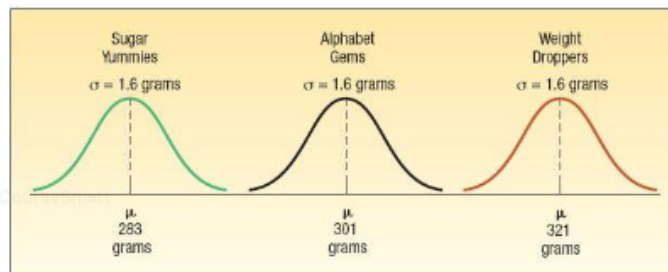


CHART 7-5 Normal Probability Distributions Having Different Means but Equal Standard Deviations





Finally, Chart 7-6 shows three normal distributions having different means and standard deviations. They show the distribution of tensile strengths, measured in pounds per square inch (psi), for three types of cables.

Unequal means, unequal standard deviations

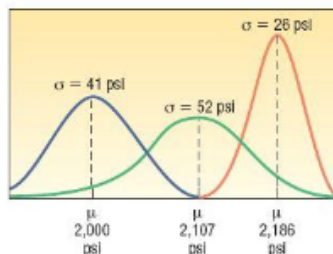


CHART 7-6 Normal Probability Distributions with Different Means and Standard Deviations

In Chapter 6, recall that discrete probability distributions show the specific likelihood a discrete value will occur. For example, in Section 6.5 on page 196, the binomial distribution is used to calculate the probability that none of the five flights arriving at the Bradford Pennsylvania Regional Airport would be late.

With a continuous probability distribution, areas below the curve define probabilities. The total area under the normal curve is 1.0. This accounts for all possible outcomes. Because a normal probability distribution is symmetric, the area under the curve to the left of the mean is 0.5, and the area under the curve to the right of the mean is 0.5. Apply this to the distribution of Sugar Yummies in Chart 7-5. It is normally distributed with a mean of 283 grams. Therefore, the probability of filling a box with more than 283 grams is 0.5 and the probability of filling a box with less than 283 grams is 0.5. We can also determine the probability that a box weighs between 280 and 286 grams. However, to determine this probability we need to know about the standard normal probability distribution.

7.4 The Standard Normal Probability Distribution

The number of normal distributions is unlimited, each having a different mean (μ), standard deviation (σ), or both. While it is possible to provide probability tables for discrete distributions such as the binomial and the Poisson, providing tables for the infinite number of normal distributions is impossible. Fortunately, one member of the family can be used to determine the probabilities for all normal probability distributions. It is called the **standard normal probability distribution**, and it is unique because it has a mean of 0 and a standard deviation of 1.

Any normal probability distribution can be converted into a standard normal probability distribution by subtracting the mean from each observation and dividing this difference by the standard deviation. The results are called **z values** or **z scores**.

There is only one standard normal distribution. It has a mean of 0 and a standard deviation of 1.

z VALUE The signed distance between a selected value, designated X , and the mean, μ , divided by the standard deviation, σ .

So, a z value is the distance from the mean, measured in units of the standard deviation.

In terms of a formula:

STANDARD NORMAL VALUE

$$z = \frac{X - \mu}{\sigma}$$

[7-5]

LO4 Convert a normal distribution to the standard normal distribution.



**Statistics in Action**

An individual's skills depend on a combination of many hereditary and environmental factors, each having about the same amount of weight or influence on the skills. Thus, much like a binomial distribution with a large number of trials, many skills and attributes follow the normal distribution. For example, scores on the Scholastic Aptitude Test (SAT) are normally distributed with a mean of 1000 and a standard deviation of 140.

LO5 Find the probability that a normally distributed random variable is between two values

where:

X is the value of any particular observation or measurement.

μ is the mean of the distribution.

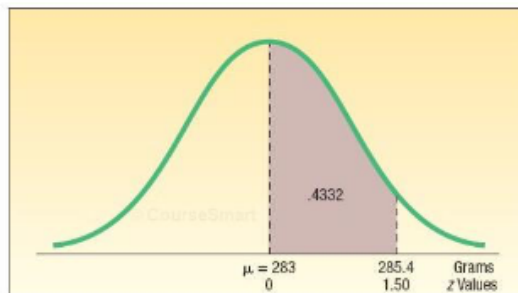
σ is the standard deviation of the distribution.

As noted in the preceding definition, a z value expresses the distance or difference between a particular value of X and the arithmetic mean in units of the standard deviation. Once the normally distributed observations are standardized, the z values are normally distributed with a mean of 0 and a standard deviation of 1. So the z distribution has all the characteristics of any normal probability distribution. These characteristics are listed on page 227. The table in Appendix B.1 (also on the inside back cover) lists the probabilities for the standard normal probability distribution. A small portion of this table follows.

TABLE 7-1 Areas under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	...
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	
.							
.							
.							

To explain, suppose we wish to compute the probability that boxes of Sugar Yummies weigh between 283 and 285.4 grams. From Chart 7-5, we know that the box weight of Sugar Yummies follows the normal distribution with a mean of 283 grams and a standard deviation of 1.6 grams. We want to know the probability or area under the curve between the mean, 283 grams, and 285.4 grams. We can also express this problem using probability notation, similar to the style used in the previous chapter: $P(283 < \text{weight} < 285.4)$. To find the probability, it is necessary to convert both 283 grams and 285.4 grams to z values using formula (7-5). The z value corresponding to 283 is 0, found by $(283 - 283)/1.6$. The z value corresponding to 285.4 is 1.50 found by $(285.4 - 283)/1.6$. Next we go to the table in Appendix B.1. A portion of the table is repeated as Table 7-1. Go down the column of the table headed by the letter z to 1.5. Then move horizontally to the right and read the probability under the column headed 0.00. It is 0.4332. This means the area under the curve between 0.00 and 1.50 is 0.4332. This is the probability that a randomly selected box of Sugar Yummies will weigh between 283 and 285.4 grams. This is illustrated in the following graph.





Applications of the Standard Normal Distribution

What is the area under the curve between the mean and X for the z values in Table 7-2 below? Check your answers against those given. You will need to use Appendix B.1 or the table located on the inside back cover of the text.

TABLE 7-2 Areas for Selected Values of z

Selected z Values	Area
2.84	.4977
1.00	.3413
0.49	.1879

Now we will compute the z value given the population mean, μ , the population standard deviation, σ , and a selected X .

Example

The weekly incomes of shift foremen in the glass industry follow the normal probability distribution with a mean of \$1,000 and a standard deviation of \$100. What is the z value for the income, let's call it X , of a foreman who earns \$1,100 per week? For a foreman who earns \$900 per week?

Solution

Using formula (7-5), the z values for the two X values (\$1,100 and \$900) are:

$$\begin{aligned} \text{For } X = \$1,100: \quad z &= \frac{X - \mu}{\sigma} \\ &= \frac{\$1,100 - \$1,000}{\$100} \\ &= 1.00 \end{aligned} \qquad \begin{aligned} \text{For } X = \$900: \quad z &= \frac{X - \mu}{\sigma} \\ &= \frac{\$900 - \$1,000}{\$100} \\ &= -1.00 \end{aligned}$$

The z of 1.00 indicates that a weekly income of \$1,100 is one standard deviation above the mean, and a z of -1.00 shows that a \$900 income is one standard deviation below the mean. Note that both incomes (\$1,100 and \$900) are the same distance (\$100) from the mean.

Self-Review 7-2



Using the information in the preceding example ($\mu = \$1,000$, $\sigma = \$100$), convert:

- The weekly income of \$1,225 to a z value.
- The weekly income of \$775 to a z value.

The Empirical Rule

Before examining further applications of the standard normal probability distribution, we will consider three areas under the normal curve that will be used extensively in the following chapters. These facts were called the Empirical Rule in Chapter 3 (see page 86).

LO6 Find probabilities using the Empirical Rule.

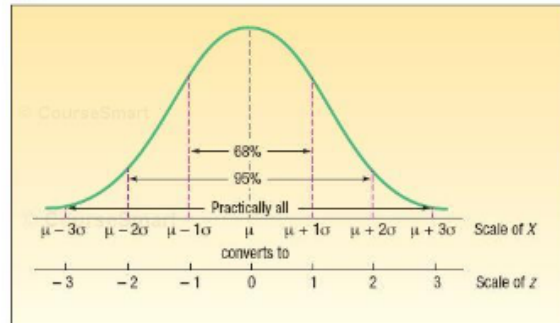
- About 68 percent of the area under the normal curve is within one standard deviation of the mean. This can be written as $\mu \pm 1\sigma$.
- About 95 percent of the area under the normal curve is within two standard deviations of the mean, written $\mu \pm 2\sigma$.
- Practically all of the area under the normal curve is within three standard deviations of the mean, written $\mu \pm 3\sigma$.





This information is summarized in the following graph.

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Transforming measurements to standard normal deviates changes the scale. The conversions are also shown in the graph. For example, $\mu + 1\sigma$ is converted to a z value of 1.00. Likewise, $\mu - 2\sigma$ is transformed to a z value of -2.00 . Note that the center of the z distribution is zero, indicating no deviation from the mean, μ .

Example

As part of its quality assurance program, the Autolite Battery Company conducts tests on battery life. For a particular D-cell alkaline battery, the mean life is 19 hours. The useful life of the battery follows a normal distribution with a standard deviation of 1.2 hours. Answer the following questions.

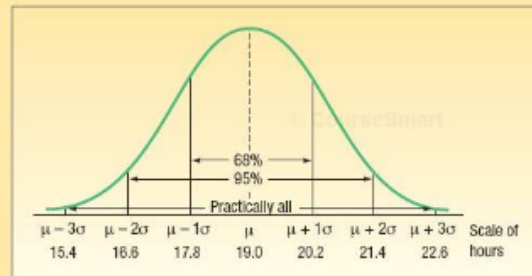
1. About 68 percent of the batteries failed between what two values?
2. About 95 percent of the batteries failed between what two values?
3. Virtually all of the batteries failed between what two values?

Solution

We can use the results of the Empirical Rule to answer these questions.

1. About 68 percent of the batteries will fail between 17.8 and 20.2 hours, found by $19.0 \pm 1(1.2)$ hours.
2. About 95 percent of the batteries will fail between 16.6 and 21.4 hours, found by $19.0 \pm 2(1.2)$ hours.
3. Practically all failed between 15.4 and 22.6 hours, found by $19.0 \pm 3(1.2)$ hours.

This information is summarized on the following chart.



**Self-Review 7-3**

The distribution of the annual incomes of a group of middle-management employees at Compton Plastics approximates a normal distribution with a mean of \$47,200 and a standard deviation of \$600.

- About 68 percent of the incomes lie between what two amounts?
- About 95 percent of the incomes lie between what two amounts?
- Virtually all of the incomes lie between what two amounts?
- What are the median and the modal incomes?
- Is the distribution of incomes symmetrical?

Exercises**connect**

- Explain what is meant by this statement: "There is not just one normal probability distribution but a 'family' of them."
- List the major characteristics of a normal probability distribution.
- The mean of a normal probability distribution is 500; the standard deviation is 10.
 - About 68 percent of the observations lie between what two values?
 - About 95 percent of the observations lie between what two values?
 - Practically all of the observations lie between what two values?
- The mean of a normal probability distribution is 60; the standard deviation is 5.
 - About what percent of the observations lie between 55 and 65?
 - About what percent of the observations lie between 50 and 70?
 - About what percent of the observations lie between 45 and 75?
- The Kamp family has twins, Rob and Rachel. Both Rob and Rachel graduated from college 2 years ago, and each is now earning \$50,000 per year. Rachel works in the retail industry, where the mean salary for executives with less than 5 years' experience is \$35,000 with a standard deviation of \$8,000. Rob is an engineer. The mean salary for engineers with less than 5 years' experience is \$60,000 with a standard deviation of \$5,000. Compute the z values for both Rob and Rachel and comment on your findings.
- A recent article in the *Cincinnati Enquirer* reported that the mean labor cost to repair a heat pump is \$90 with a standard deviation of \$22. Monte's Plumbing and Heating Service completed repairs on two heat pumps this morning. The labor cost for the first was \$75 and it was \$100 for the second. Assume the distribution of labor costs follows the normal probability distribution. Compute z values for each and comment on your findings.

Finding Areas under the Normal Curve

The next application of the standard normal distribution involves finding the area in a normal distribution between the mean and a selected value, which we identify as X . The following example will illustrate the details.

Example

Recall in an earlier example (see page 231) we reported that the mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. That is, $\mu = \$1,000$ and $\sigma = \$100$. What is the likelihood of selecting a foreman whose weekly income is between \$1,000 and \$1,100? We write this question in probability notation as: $P(\$1,000 < \text{weekly income} < \$1,100)$.

Solution

We have already converted \$1,100 to a z value of 1.00 using formula (7-5). To repeat:

$$z = \frac{X - \mu}{\sigma} = \frac{\$1,100 - \$1,000}{\$100} = 1.00$$





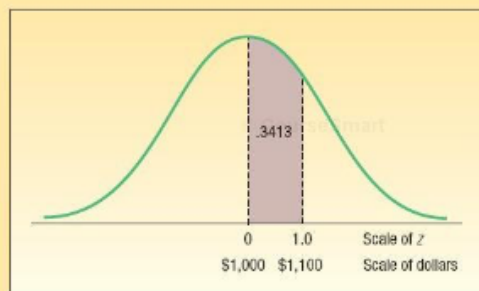
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The probability associated with a z of 1.00 is available in Appendix B.1. A portion of Appendix B.1 follows. To locate the probability, go down the left column to 1.0, and then move horizontally to the column headed .00. The value is .3413.

z	0.00	0.01	0.02
.	.	.	.
.	.	.	.
0.7	.2580	.2611	.2642
0.8	.2881	.2910	.2939
0.9	.3159	.3186	.3212
1.0	.3413	.3438	.3461
1.1	.3643	.3665	.3686
.	.	.	.
.	.	.	.
.	.	.	.

The area under the normal curve between \$1,000 and \$1,100 is .3413. We could also say 34.13 percent of the shift foremen in the glass industry earn between \$1,000 and \$1,100 weekly, or the likelihood of selecting a foreman and finding his or her income is between \$1,000 and \$1,100 is .3413.

This information is summarized in the following diagram.

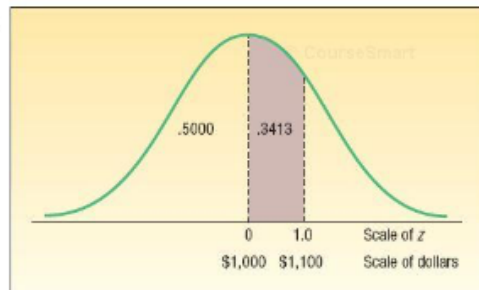


In the example just completed, we are interested in the probability between the mean and a given value. Let's change the question. Instead of wanting to know the probability of selecting at random a foreman who earned between \$1,000 and \$1,100, suppose we wanted the probability of selecting a foreman who earned less than \$1,100. In probability notation, we write this statement as $P(\text{weekly income} < \$1,100)$. The method of solution is the same. We find the probability of selecting a foreman who earns between \$1,000, the mean, and \$1,100. This probability is .3413. Next, recall that half the area, or probability, is above the mean and half is below. So the probability of selecting a foreman earning less than \$1,000 is .5000. Finally, we add the two probabilities, so $.3413 + .5000 = .8413$. About 84 percent of the foremen in the glass industry earn less than \$1,100 per week. See the following diagram.

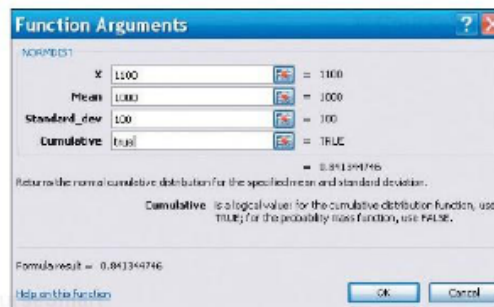


**Statistics in Action**

Many processes, such as filling soda bottles and canning fruit, are normally distributed. Manufacturers must guard against both over- and underfilling. If they put too much in the can or bottle, they are giving away their product. If they put too little in, the customer may feel cheated and the government may question the label description. "Control charts," with limits drawn three standard deviations above and below the mean, are routinely used to monitor this type of production process.



Excel will calculate this probability. The necessary commands are in the **Software Commands** section at the end of the chapter. The answer is .8413, the same as we calculated.

**Example**

Refer to the information regarding the weekly income of shift foremen in the glass industry. The distribution of weekly incomes follows the normal probability distribution, with a mean of \$1,000 and a standard deviation of \$100. What is the probability of selecting a shift foreman in the glass industry whose income is:

- Between \$790 and \$1,000?
- Less than \$790?

Solution

We begin by finding the z value corresponding to a weekly income of \$790. From formula (7-5):

$$z = \frac{X - \mu}{s} = \frac{\$790 - \$1,000}{\$100} = -2.10$$

See Appendix B.1. Move down the left margin to the row 2.1 and across that row to the column headed 0.00. The value is .4821. So the area under the standard normal curve corresponding to a z value of 2.10 is .4821. However, because the normal distribution is symmetric, the area between 0 and a negative z value is the same as that between 0 and the corresponding positive z value. The likelihood of finding a foreman earning between \$790 and \$1,000 is .4821. In probability notation, we write $P(\$790 < \text{weekly income} < \$1000) = .4821$.



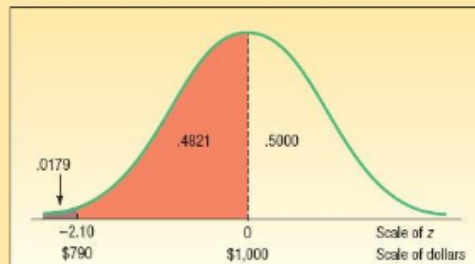


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z	0.00	0.01	0.02
.	.	.	.
.	.	.	.
.	.	.	.
2.0	.4772	.4778	.4783
2.1	.4821	.4826	.4830
2.2	.4861	.4864	.4868
2.3	.4893	.4896	.4898
.	.	.	.
.	.	.	.
.	.	.	.

The mean divides the normal curve into two identical halves. The area under the half to the left of the mean is .5000, and the area to the right is also .5000. Because the area under the curve between \$790 and \$1,000 is .4821, the area below \$790 is .0179, found by $.5000 - .4821$. In probability notation, we write $P(\text{weekly income} < \$790) = .0179$.

This means that 48.21 percent of the foremen have weekly incomes between \$790 and \$1,000. Further, we can anticipate that 1.79 percent earn less than \$790 per week. This information is summarized in the following diagram.



Self-Review 7-4



The temperature of coffee sold at the Coffee Bean Cafe follows the normal probability distribution, with a mean of 150 degrees. The standard deviation of this distribution is 5 degrees.

- What is the probability that the coffee temperature is between 150 degrees and 154 degrees?
- What is the probability that the coffee temperature is more than 164 degrees?

Exercises

connect

- A normal population has a mean of 20.0 and a standard deviation of 4.0.
 - Compute the z value associated with 25.0.
 - What proportion of the population is between 20.0 and 25.0?
 - What proportion of the population is less than 18.0?
- A normal population has a mean of 12.2 and a standard deviation of 2.5.
 - Compute the z value associated with 14.3.
 - What proportion of the population is between 12.2 and 14.3?
 - What proportion of the population is less than 10.0?





15. A recent study of the hourly wages of maintenance crew members for major airlines showed that the mean hourly salary was \$20.50, with a standard deviation of \$3.50. Assume the distribution of hourly wages follows the normal probability distribution. If we select a crew member at random, what is the probability the crew member earns:
- Between \$20.50 and \$24.00 per hour?
 - More than \$24.00 per hour?
 - Less than \$19.00 per hour?
16. The mean of a normal probability distribution is 400 pounds. The standard deviation is 10 pounds.
- What is the area between 415 pounds and the mean of 400 pounds?
 - What is the area between the mean and 395 pounds?
 - What is the probability of selecting a value at random and discovering that it has a value of less than 395 pounds?

Another application of the normal distribution involves combining two areas, or probabilities. One of the areas is to the right of the mean and the other to the left.

Example

Recall the distribution of weekly incomes of shift foremen in the glass industry. The weekly incomes follow the normal probability distribution, with a mean of \$1,000 and a standard deviation of \$100. What is the area under this normal curve between \$840 and \$1,200?

Solution

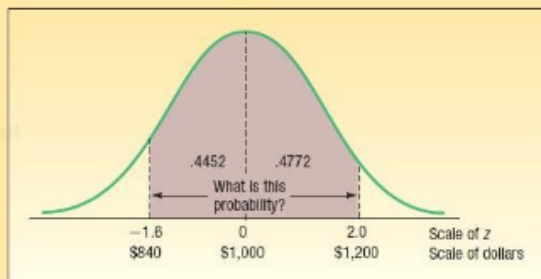
The problem can be divided into two parts. For the area between \$840 and the mean of \$1,000:

$$z = \frac{\$840 - \$1,000}{\$100} = \frac{-\$160}{\$100} = -1.60$$

For the area between the mean of \$1,000 and \$1,200:

$$z = \frac{\$1,200 - \$1,000}{\$100} = \frac{\$200}{\$100} = 2.00$$

The area under the curve for a z of -1.60 is .4452 (from Appendix B.1). The area under the curve for a z of 2.00 is .4772. Adding the two areas: $.4452 + .4772 = .9224$. Thus, the probability of selecting an income between \$840 and \$1,200 is .9224. In probability notation, we write $P(\$840 < \text{weekly income} < \$1,200) = .4452 + .4772 = .9224$. To summarize, 92.24 percent of the foremen have weekly incomes between \$840 and \$1,200. This is shown in a diagram:



Another application of the normal distribution involves determining the area between values on the same side of the mean.

